Geometrically Parameterized Interconnect Performance Models for Interconnect Synthesis*

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ABSTRACT

In this paper we describe an approach for generating geometrically-parameterized integrated-circuit interconnect models that are efficient enough for use in interconnect synthesis. The model generation approach presented is automatic, and is based on a multi-parameter model-reduction algorithm. The effectiveness of the technique is tested using a multi-line bus example, where both wire spacing and wire width are considered as geometric parameters. Experimental results demonstrate that the generated models accurately predict both delay and cross-talk effects over a wide range of spacing and width variation.

1. INTRODUCTION

Developers of routing tools for mixed signal applications could make productive use of more accurate performance models for interconnect, but the cost of extracting even a modestly accurate model for a candidate route is far beyond the computational budget of the inner loop of a router. If it were possible to extract geometrically parameterized models of interconnect performance, then such models could be used for detailed interconnect synthesis in performance critical digital or analog applications. In this paper we present a scheme for automatically constructing parameterized models for interconnect, and demonstrate the scheme's effectiveness using a width and spacing parameterized multi-line bus.

The idea of generating parameterized reduced-order interconnect models is not new, recent approaches have been developed that focus on statistical performance evaluation [1, 2] and clock skew minimization [3]. Our work differs from the cited efforts in two important ways. First, the target application, interconnect synthesis, requires parameterized models valid over a wide geometric range. Second, the technique described below is a multi-parameter extension of the projection-subspace based moment matching methods that have proved so effective in interconnect modeling [12, 13, 10, 9, 8, 7, 11].

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In the following section we present the basic background on multi-parameter model-order reduction for a two-parameter case, and then in section three we describe the generalization to an arbitrary number of parameters. In section four, we demonstrate the effectiveness of the method on a wire-spacing parameterized multi-line bus example, and consider both delay and cross-talk effects. In section five we use the generalized multi-parameter model reduction approach to re-examine the multi-line bus example, but now allow both wire width and wire spacing to be parameters. Conclusions are given in section six.

2. BACKGROUND

One recently developed technique for generating simple geometrically parameterized models of physical systems is based on first using a very detailed representation, such as a discretized partial differential equation, and then reducing that representation while preserving the variation due to changing parameters [5]. The reduction approach used for handling geometric parameter variation in these physical system closely parallels the techniques for dynamical system model reduction, a situation that follows from considering the Laplace transform description of a dynamical system and then allowing the frequency variable to substitute for a geometric parameter. This close parallelism has allowed for some cross-fertilization, for example a subspace-projection based moment matching method was borrowed from the dynamical system model-reduction context and used to automatically generate spacing-parameterized models of wire capacitances [6].

The observation that geometric parameters and frequency variables are interchangeable, at least in a restricted setting, suggests that the problem of generating geometrically parameterized reduced-order models of interconnect can be formulated as a multi-parameter model-order reduction problem. In addition, it is possible to exploit the recently developed connection between projection subspaces and multi-parameter moment-matching [4] to generate an effective algorithm. Below, we make this idea more precise.

Consider the linear system

$$[s_1E_1 + s_2E_2 - A]x = Bu \qquad (1)$$
$$y = Cx \qquad (2)$$

where s_1 and s_2 are scalar parameters; x is a state vector of dimension n; u and y are m-dimensional input and output vectors; E_1, E_2 and A are $n \times n$ matrices; and B and C are $n \times m$ and $m \times n$ matrices which define how the inputs and outputs relate to the state vector x.

If one of the parameters, s_1 or s_2, are associated with frequency, and the other associated with a geometric variation, then (1) would be a dynamical system and $E(s_1,s_2) = s_1E_1 + s_2E_2 - A$ would be its descriptor matrix.

For many interconnect problems, the number of inputs and outputs, m, is typically much smaller than n, the number of states needed to accurately represent the electrical behavior of the interconnect. In order to generate a representation of the input-output behavior given by (1) using many fewer states, one can use a projection approach [7]. In the projection approach, one first constructs an $n \times q$ projection matrix V where $q \ll n$, and then one generates the reduced model from the matrices of the original system using congruence transformations [10]. Specifically, the reduced system is given by

$$\begin{aligned} [s_1V^TE_1V + s_2V^TE_2V - V^TAV]\hat{x} &= V^TBu && (3) \\ y &= CV\hat{x} && (4) \end{aligned}$$

were the reduced state vector $\hat{x}$ is of dimension q and is representing the projection of the large original state vector $x \approx V\hat{x}$.

The columns of V are typically chosen in such a way that the final response of the reduced system matches q terms in the Taylor series expansion in s_1 and s_2 of the original response. For a nonsingular A we can write (1) as

$$\begin{aligned} [I - (s_1M_1 + s_2M_2)]x &= B_Mu \\ y &= Cx \end{aligned}$$

where

$$\begin{aligned} M_1 &= -A^{-1}E_1 \\ M_2 &= -A^{-1}E_2 \\ B_M &= -A^{-1}B. \end{aligned}$$

We can then derive an expression for the state vector x which we can conveniently expand in Taylor series

$$\begin{aligned} x &= [I - (s_1M_1 + s_2M_2)]^{-1}B_M\, u \\ &= \sum_{m=0}^{\infty} [s_1M_1 + s_2M_2]^m B_M\, u \\ &= \sum_{m=0}^{\infty}\sum_{k=0}^{m} F_k^m(M_1,M_2)B_M\, u\ \ s_1^{m-k}s_2^k \end{aligned}$$

The coefficients of the series $F_k^m(M_1,M_2)$ can be calculated using [4]

$$F_k^m(M_1,M_2) = \\ = \begin{cases} 0 & \text{if } k \notin \{0,1,\dots,m\} \\ I & \text{if } m = 0 \\ M_1F_k^{m-1}(M_1,M_2) + M_2F_{k-1}^{m-1}(M_1,M_2) & \text{otherwise} \end{cases}$$

In [4] it is also shown that for a single input system ($B_M = b$) if the columns of V are constructed to span the Krylov subspace

$$V = \text{colspan}\{b,\ M_1b,\ M_2b,\ M_1^2b,\ (M_1M_2 + M_2M_1)b,\ M_2^2b,\ \dots\},$$

or equivalently,

$$V = \text{colspan}\left\{ \bigcup_{m=0}^{n_q} \left(\bigcup_{k=0}^{m} F_k^m(M_1,M_2)b \right) \right\},$$

then the reduced model matches the first $q = n_q(n_q+1)/2$ moments of the Taylor series expansion in s_1 and s_2.

3. P-PARAMETERS MODEL ORDER REDUCTION

In this Section we consider the extension of the previous results to a linear system

$$\begin{aligned} [s_1E_1 + \dots + s_pE_p - A]x &= Bu && (5) \\ y &= Cx && (6) \end{aligned}$$

where the descriptor matrix $E(s_1,\dots,s_p) = s_1E_1 + \dots + s_pE_p - A$ depends on p parameters $s_1,\dots,s_p$. The reduced model can still be generated using a congruence transformation

$$\begin{aligned} [s_1V^TE_1V + \dots + s_pV^TE_pV - V^TAV]\hat{x} &= V^TBu \\ y &= CV\hat{x} \end{aligned}$$

and once again, in order to calculate the column span of the projection matrix V it is convenient to write the system (5) as

$$\begin{aligned} [I - (s_1M_1 + \dots + s_pM_p)]x &= B_Mu \\ y &= Cx \end{aligned}$$

where

$$\begin{aligned} M_i &= -A^{-1}E_i \quad \text{for} \quad i = 1,2,\dots,p \\ B_M &= -A^{-1}B \end{aligned}$$

and expanding in Taylor series

$$\begin{aligned} x &= [I - (s_1M_1 + \dots + s_pM_p)]^{-1}B_M\, u \\ &= \sum_{m=0}^{\infty} [s_1M_1 + \dots + s_pM_p]^m B_M\, u \\ &= \sum_{m=0}^{\infty} \sum_{k_2=0}^{m-(k_3+\dots+k_p)} \dots \\ &\quad \dots \sum_{k_{p-1}=0}^{m-k_p} \sum_{k_p=0}^{m} [F_{k_2,\dots,k_p}^m(M_1,\dots,M_p)B_M\, u]\ \ s_1^{m-(k_2+\dots+k_p)}s_2^{k_2}\dots s_p^{k_p} \end{aligned}$$

The coefficients of the series $F_{k_2,\dots,k_p}^m(M_1,\dots,M_p)$ can be calculated using:

$$F_{k_2,\dots,k_p}^m(M_1,\dots,M_p) = \begin{cases} 0 & \text{if } k_i \notin \{0,1,\dots,m\} \quad i = 2,\dots,p \\ 0 & \text{if } k_2 + \dots + k_p \notin \{0,1,\dots,m\} \\ I & \text{if } m = 0 \end{cases}$$

and for all other cases

$$\begin{aligned} F_{k_2,\dots,k_p}^m(M_1,\dots,M_p) &= M_1F_{k_2,\dots,k_p}^{m-1}(M_1,\dots,M_p) + && (7) \\ &\quad + M_2F_{k_2-1,\dots,k_p}^{m-1}(M_1,\dots,M_p) + \dots \\ &\quad + M_pF_{k_2,\dots,k_p-1}^{m-1}(M_1,\dots,M_p) \end{aligned}$$

For a single input system ($B_M = b$) the columns of V can be constructed to span the Krylov subspace

$$\begin{aligned} V &= \text{colspan}\{b, M_1b, M_2b, \dots, M_pb, M_1^2b, (M_1M_2 + M_2M_1)b, \dots, \\ &\quad \dots, (M_1M_p + M_pM_1)b, M_2^2b, (M_2M_3 + M_2M_3)b, \dots\}, \end{aligned}$$

or equivalently

$$V = \text{colspan}\left\{ \bigcup_{m=0}^{n_q} \bigcup_{k_2=0}^{m-(k_p+\dots+k_3)} \dots \bigcup_{k_{p-1}=0}^{m-k_p} \bigcup_{k_p=0}^{m} F_{k_2,\dots,k_p}^m(M_1,\dots,M_p)b \right\}.$$

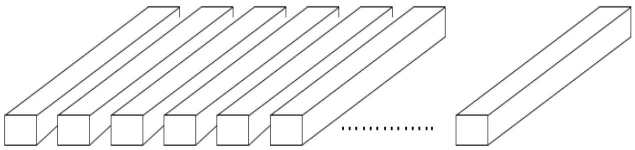

Figure 1: Sketch of the modeled 16 parallel wires interconnect bus.

For a multi-input system the columns of V can then be constructed to span the Krylov subspaces produced by the columns of B_M

$$V = \text{colspan}\left\{ \begin{array}{l} \bigcup_{m=0}^{n_q} \bigcup_{k_2=0}^{m-(k_p+\ldots k_3)} \cdots \bigcup_{k_p=0}^{m} F^m_{k_2,\ldots,k_p}(M_1,\cdots,M_p)[B_M]_1, \cdots, \\ \bigcup_{m=0}^{n_q} \bigcup_{k_2=0}^{m-(k_p+\ldots k_3)} \cdots \bigcup_{k_p=0}^{m} F^m_{k_2,\ldots,k_p}(M_1,\cdots,M_p)[B_M]_j \end{array} \right\}$$

4. EXAMPLE: A BUS MODEL PARAMETRIZED IN THE WIRES' SPACING

One design consideration for interconnect busses is the trade-off between:

- wider spacing to reduce propagation delays and crosstalk
- narrower spacing to reduce area and therefore cost.

In this example we have used a multi-parameter model order reduction approach to construct a low-order model of an interconnect bus, parametrized by the wire spacing. The model can be efficiently constructed "on the fly" during the design and can account for the topology of the surrounding interconnect already present in the design. Once produced, the model can be simply evaluated for different values of the main parameter, the wire spacing, in order to determine propagation delay, crosstalk or even detailed step responses.

Our example problem is the bus in Fig. 1 which consists of $N = 16$ parallel wires, with thickness $h = 1.2\mu$m, and width $w = 1\mu$m. The total length of each wire is $l = 1$mm. Above and below our bus we assumed a random collection of interconnect at several layout levels ranging from a distance of 1μm to 5μm. We have subdivided each wire into 20 equal sections delimited by $n = 21$ nodes. Each section has been modeled with a resistor. Each node has a "grounded capacitor" representing the interaction with upper and lower interconnect levels. In addition, each node has two coupling capacitors to the adjacent wires on the bus. The value of the capacitors was determined using simple parallel plate formulas. Standard frequency domain nodal analysis leads to a system of equations of the form

$$\begin{aligned} s\left(C_g + \frac{C_s}{d}\right) v(s) + Gv(s) &= Bv_{in}(s) \qquad (8) \\ v_{out}(s) &= Cv(s), \qquad (9) \end{aligned}$$

where s is the Laplace Transform variable, d is the spacing between wires, G is the $n \times n$ nodal conductance matrix, The $n \times n$ matrix C_g is the diagonal nodal matrix associated with the grounded capacitors, and C_s is the sparse nodal matrix associated with the adjacent coupling capacitors. B is the $n \times m$ matrix relating m input voltages v_{in} to the n internal node potentials v, C is a $m \times n$ matrix relating node potentials v to the m output voltages v_{out}. For simplicity in this example we assumed all wires are driven by sources having the same impedance $r_d = 1/g_d$. In general when g_d is small compared to the wire conductance, all the capacitors in the different sections of each wire appear as lumped, and the detailed model presented here is not necessary. A more interesting case is observed when instead g_d is large. In such case the wires charge up slowly from the input side of the bus and continue to charge up along the length of the bus. In order to observe this more interesting effect we chose $g_d = g$ where g is the conductance of each of the 20 sections in each wire. All the wires are left open on the other side.

4.1 Crosstalk from one input to all outputs

To determine the crosstalk generated on all the outputs from a transition on a single input, the input matrix becomes a unit vector,

$$B = b = [0\ldots 0\, g_d\, 0\ldots 0]^T,$$

and the output matrix becomes a set of m unit vectors

$$C = \begin{bmatrix} \ldots 010\ldots & & & \\ & \ldots 010\ldots & & \\ & & \ddots & \\ & & & \ldots 01 \end{bmatrix}$$

The system in (8) can be reduced in the form (1) shown above in Section 2 by defining

$$\begin{aligned} s_1 &= s \\ s_2 &= \frac{s}{d} \end{aligned}$$

The problem is better parameterized using the change of variables $\gamma = 1/d$ and then using a Taylor series expansion around a nominal spacing value d_0

$$\gamma = \gamma_0 + \Delta\gamma = \frac{1}{d_0 + \Delta d} = \frac{1}{d}$$

so that (8) becomes

$$s\left[C_g + C_s(\gamma_0 + \Delta\gamma)\right] v + Gv = bv_{in}$$

or

$$[s(C_g + C_s\gamma_0) + s\Delta\gamma C_s + G]v = bv_{in}$$

which can be reorganized to the form in (1) using

$$\begin{aligned} E_1 &= C_g + C_s\gamma_0 \\ E_2 &= C_s \\ A &= -G \\ s_1 &= s \\ s_2 &= s\Delta\gamma \end{aligned}$$

The original system for this example has order 336 (16 wires $\times$ 21 nodes each). We performed a model order reduction procedure as described in Section 2 and obtained a small model capturing the transfer functions from one input to all outputs.

$$\begin{aligned} [I - s(\hat{M}_{1,r} + \Delta\gamma \hat{M}_{2,r})]\hat{v} &= \hat{b}v_{in} \qquad (10) \\ v_{out} &= \hat{C}\hat{v}, \qquad (11) \end{aligned}$$

where

$$\begin{aligned} \hat{M}_{1,r} &= V^T M_1 V = V^T A^{-1} E_1 V = -V^T G^{-1}(C_g + C_s\gamma_0)V \\ \hat{M}_{2,r} &= V^T M_2 V = V^T A^{-1} E_2 V = -V^T G^{-1} C_s V \\ \hat{b} &= V^T A^{-1} b \\ \hat{C} &= VC. \end{aligned}$$

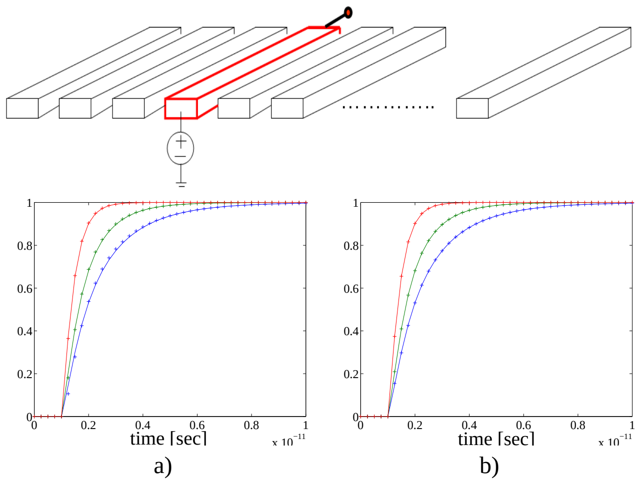

Figure 2: Responses at the end of wire 4 when a step is applied at the beginning of the same wire. Continuous lines are the response of the original system (order 336). Small crosses are the response of the reduced model, order 3 on the left, and order 6 on the right. The model was constructed using a nominal wire spacing $d_0 = 1um$ and responses are shown here evaluating it at spacings (from the lowest curves to the highest) $d = d_0 + \Delta d = 0.5\mu m, 1\mu m, 10\mu m$.

The step response at the output at the end of the input wire is shown in Fig 2.a comparing the step responses of the original system (continuous lines) and a reduced model of order three (small crosses) when the spacing distance assumes the values $d = d_0 + \Delta d = 0.5\mu m, 1\mu m, 10\mu$m. The model was constructed using a nominal spacing $d_0 = 1\mu$m, hence the error is smallest for $d \approx d_0 = 1\mu$m. Figure 2.b shows the same comparison with a reduced model of order six. One can notice that the reduced model can be easily and accurately used to evaluate the step response and propagation delay for any value of parameter d by simply calculating

$$\Delta\gamma = \frac{1}{d} - \frac{1}{d_0}$$

and then plugging into the reduced model (10). From the reduced model (10) we have readily available not only step responses on the same wire, but also crosstalk step responses from one wire to all the other wires. Fig. 3.a shows for instance step responses from the input of wire 4 to the output of wires 4, 5, 6 and 7. In this figure we compare again the response of the original system order 336 (continues curves) with the response of a reduced model order 10 (small crosses) constructed at nominal spacing $d_0 = 1\mu$m, but evaluated in this particular figure at spacing $d = 0.5\mu$m. Note that the model produced by our procedure is parametrized in the wire spacing, hence *any* of such crosstalk responses can be evaluated at *any* spacing. For instance we show in Fig. 3.b the response at the output of wire 5 when a step waveform is applied at the input of wire 4 for different spacing values, $d = d_0 + \Delta d = 0.5\mu m, 1\mu m, 10\mu m$.

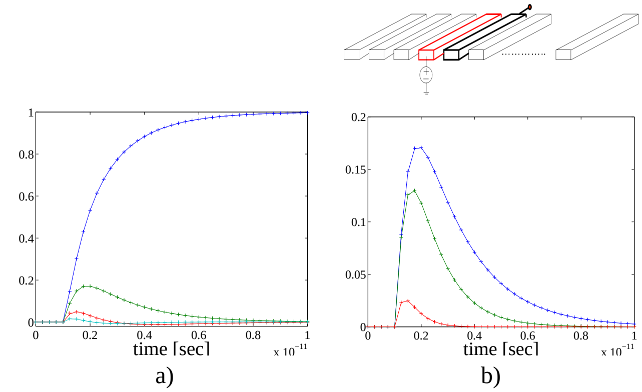

Figure 3: On the left: responses at the end of wires (from highest to lowest curve) 4, 5, 6 and 7 when a step is applied at the beginning of wire 4. Continuous lines are the response of the original system (order 336). Small crosses are the response of the reduced model (order 10). The model was constructed using a nominal wire spacing $d_0 = 1um$ and responses are shown here evaluating it at spacing $d = 0.5\mu m$. On the right: crosstalk responses at the end of wire 5 when a step is applied at the beginning of wire 4, for different values of spacing (from highest to lowest curve) $d = d_0 + \Delta d = 0.5\mu m, 1\mu m, 10\mu m$.

4.2 Exploiting the adjoint method for crosstalk from all inputs to one output

It is possible to construct with the same amount of calculation a model that provides the susceptibility of one output to all inputs. In order to do this we can use an adjoint method and start from an original system which swaps positions of C and B and transposes all system matrices

$$\left[I - (s_1 M_1^T + s_2 M_2^T)\right] v' = c^T v'_{in} \qquad (12)$$

$$v'_{out} = B_M^T v', \qquad (13)$$

In this case the columns of the projection operator V will span the Krylov subspace

$$\begin{aligned} V' = \text{colspan}\{ \; & c^T, M_1^T c^T, M_2^T c^T, M_1^T M_1^T c^T, \\ & (M_1^T M_2^T + M_2^T M_1^T) c^T, M_2^T M_2^T c^T, \ldots \} \end{aligned}$$

or generally

$$V' = \text{colspan}\left\{ \bigcup_{m=0}^{n_q} \left(\bigcup_{k=0}^{m} F_k^m (M_1^T, M_2^T) c^T \right) \right\}.$$

In Fig. 4 we show the responses at the end of wire 4 when a step is applied at the beginning of wires 4, 5, 6 and 7. The model was constructed using a nominal wire spacing $d_0 = 1um$. Responses in Fig. 4.a are for $d = 0.25\mu m$. Responses in Fig. 4.b are for $d = 2\mu m$.

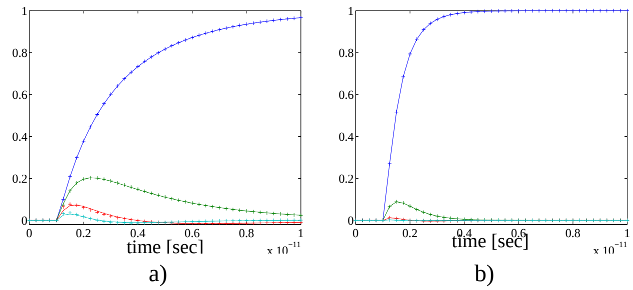

Figure 4: Responses at the end of wire 4 when a step is applied at the beginning of wires 4, 5, 6 and 7 (from highest to lowest curve). Continuous lines are the response of the original system (order 336). Small crosses are the response of the reduced model (order 10). The model was constructed using $d_0 = 1um$. Responses on the left are for $d = 0.25\mu m$, and on the right for $d = 2\mu m$.

5. EXAMPLE: BUS MODEL PARAMETRIZED IN BOTH WIRE WIDTH AND SEPARATION

Often when designing an interconnect bus, one would like to quickly evaluate design trade-offs originating not only from different wire spacings, but also for different wire widths. Wider wires have lower resistances but use more area and have higher capacitance. The higher capacitance to ground however helps improving crosstalk immunity. We show here a procedure that produces small models that can be easily evaluated with respect to propagation delays and crosstalk performance for different values of the two parameters: wire spacing $d = 1/\gamma$, and wire width W. As in the case of wire spacing, we constructed models for a given nominal wire width W_0, and then we parametrized in terms of perturbations ΔW. Considering the same bus example with N parallel wires described in Section 4, we can write the equations for the original large parametrized linear system

$$\begin{aligned} s[C'_g(W_0+\Delta W)+C_s(\gamma_0+\Delta\gamma)]v+G'(W_0+\Delta W)v &= Bv_{in} \\ v_{out} &= Cv \end{aligned}$$

where $C'_g = C_g/W_0$, $G' = G/W_0$, and C_g and G are as described in Section 4. After some algebraic manipulation one can recognize a parametrized linear system as in (5) with $p = 4$ parameters by defining

$$\begin{array}{ll} E_1 = C'_g W_0 + C_s\gamma_0 & s_1 = s \\ E_2 = C_s & s_2 = s\Delta\gamma \\ E_3 = C'_g & s_3 = s\Delta W \\ E_4 = G' & s_4 = \Delta W \\ A = -G'W_0. & \end{array}$$

One can then follow the procedure in Section 3 and construct a projection operator V. Finally the produced reduced order model is

$$\begin{aligned} [I - s(\hat{M}_{1,r} + \Delta\gamma\hat{M}_{2,r} + \Delta W\hat{M}_3) - \Delta W\hat{M}_4]\hat{v} &= \hat{b}v_{in} \quad (14) \\ v_{out} &= \hat{C}\hat{v}, \quad (15) \end{aligned}$$

where

$$\begin{aligned} \hat{M}_{1,r} &= V^T M_1 V = V^T A^{-1} E_1 V = -V^T (G'W_0)^{-1}(C'_g W_0 + C_s\gamma_0)V \\ \hat{M}_{2,r} &= V^T M_2 V = V^T A^{-1} E_1 V = -V^T (G'W_0)^{-1} C_s V \\ \hat{M}_{3,r} &= V^T M_3 V = V^T A^{-1} E_1 V = -V^T (G'W_0)^{-1} C'_g V \\ \hat{M}_{4,r} &= V^T M_4 V = V^T A^{-1} E_1 V = -V^T (G'W_0)^{-1} G' V = -\frac{I}{W_0} \\ \hat{b} &= V^T A^{-1} b = -V^T (G'W_0)^{-1} b \\ \hat{C} &= VC \end{aligned}$$

In Fig. 5 we compare the step and crosstalk responses of the original system compared to the reduced and parametrized model obtained using a Krylov subspace of order $q = 15$ ($n_q = 2$). The model was constructed using a nominal spacing $d_0 = 1\mu$m and nominal wire width $W_0 = 1\mu$m. The key point is that this parameterized model can be rapidly evaluated for any value of spacing and wire width, for instance for a fast and accurate trade-off design optimization procedure.

6. CONCLUSIONS

In this paper we described an approach for generating geometrically - parameterized integrated-circuit interconnect models that are efficient enough for use in interconnect synthesis. The model generation approach presented is automatic, and is based on a multi-parameter model-reduction algorithm. The effectiveness of the technique was tested using a multi-line bus example, where both wire spacing and wire width are considered as geometric parameters. Experimental results demonstrate that the generated models accurately predict both delay and cross-talk effects over a wide range of spacing and width variation, even when a very low order model is used.

There are many issues still left to address. The multi-parameter method was tested using only resistor-capacitor interconnect models, and accuracy issues may arise when inductance is included. We also did not investigate using multipoint moment-matching, which seems like a natural choice given the range of the parameters is often known a-priori. In addition, the multi-parameter reduction method can become quite expensive when the model has a large number of parameters, so the method would not generate a very efficient model if each wire pair spacing in a 16 wire bus was treated individually. Finally, there are some interesting error bounds in [5], and these results could be applied to automatically select the reduction order.

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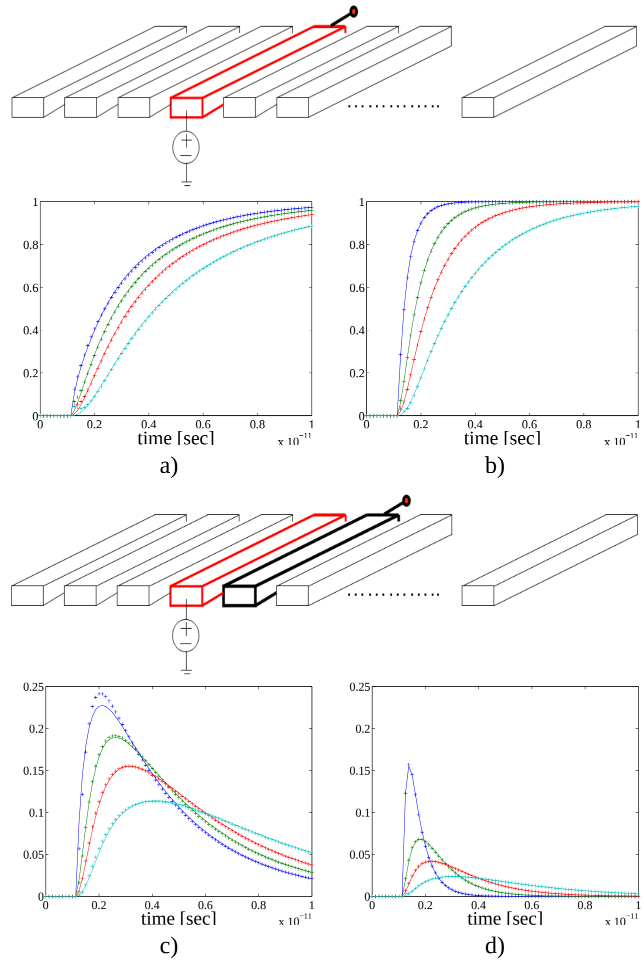

Figure 5: Original system (continuous curves) versus 15^{th} order reduced model (small crosses) using both spacing and width parameters. The nominal wire spacing was $d_0 = 1\mu$m and the nominal wire width was $W = 1\mu$m. Responses at the end of wire 4 due to a step at the beginning of the same wire are show in a) for different widths (from highest to lowest curve) $W = .25\mu$m, 2μm, 4μm, 8μm and for spacing $d = .25\mu$m. In b) we show the same responses but for spacing $d = 2\mu$m. In c) we show the crosstalk response at the end of wire 5 due to a step at the beginning of wire 4. Curves correspond to widths (from highest curve to lowest) $W = .25\mu$m, 2μm, 4μm, 8μm and spacing is $d = .25\mu$m. In d) we show the same crosstalk responses but for spacing $d = 2\mu$m.