

# Maximum Voltage Variation in the Power Distribution Network of VLSI Circuits with RLC Models\*

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## ABSTRACT

In this paper, we present a frequency-domain technique to estimate the worst-case time-domain voltage variation using RLC models for the power distribution network. The proposed method, unlike existing simulation-based techniques, can handle frequency-dependent RLC parameters and generate an upperbound on the maximum voltage drop over all possible input excitations. Pattern independent maximum envelope currents are used to estimate the upperbound on the maximum magnitude of the frequency components for the current waveform. These values are used to formulate a nonlinear optimization problem for the maximum voltage drop at nodes in the power distribution network. We then present a method to solve the nonlinear optimization problem using Lagrange multipliers. Comparisons with SPICE simulations are presented to validate the techniques presented in the paper.

## 1. INTRODUCTION

Power supply variations can change the delay of a logic gate and this can potentially change the delay of the circuit. The problem of power supply noise becomes severe in scaled technologies because of higher clock frequencies, increased current demand with significant variations, and increasing dominance of interconnect parasitics. The impact of the power supply voltage variations on circuit performance is also severe in scaled technologies because of the lower supply voltage and smaller timing margins for delay variations. A smaller timing margin implies a smaller budget for the allowable delay variations. Also, excessive supply voltage variations degrade the noise margins and in extreme cases can cause logic failures. Therefore, high-performance integrated circuits require a robust power delivery network with nominal supply voltage fluctuations. The design of such a

power distribution network requires the realization of a network that offers a small impedance to all the frequencies that can be excited by the current waveform. Hence, techniques to estimate the maximum voltage variation in the RLC power distribution network over all possible current excitations are required.

The simplest electrical model for the power distribution network is the resistive power bus model. Although it simplifies the power bus analysis, the results are accurate only if the resistance effect dominates the capacitive and inductive effects. The powerbus analysis techniques [1, 2, 3, 4], compute the IR drop at nodes in the power distribution network by using the macroblock currents as a DC current obtained heuristically, or a DC current obtained by logic simulations for a few vectors or a transient current waveform obtained by simulations for a few vectors. A resistive model for the top-cell power distribution network does not take into account the presence of significant on-chip decoupling capacitance that helps to reduce the voltage drops. In addition, ignoring the inductance can result in an underestimation of the supply voltage variations. The powerbus analysis techniques [5, 6] use the peak/ average current or the current waveform obtained by simulations for a few vectors to approximate a heuristically constructed triangular or trapezoidal macroblock current waveform. This macroblock current waveform is then used to estimate the voltage drop at nodes in the power distribution network with RLC models by simulation or by a heuristic algorithm that performs a lookup of a pre-characterized waveform library. One limitation of these time-domain simulation techniques is that the frequency-dependent resistance and inductance parameters are hard to include. At high frequencies, the resistance and inductances become frequency dependent due to phenomenon such as skin-effect and proximity-effect [7]. Accurate electrical models for the power distribution network that are valid at all frequencies are required to capture the worst-case instantaneous voltage variations. The proposed technique uses RLC models for the power distribution network and it can handle frequency dependence for any or all of the parameters.

The input pattern dependence of the current drawn by logic gates makes the problem of estimating the maximum current or maximum voltage drop a hard problem. Simulation of the circuit for a few input vectors does not guarantee the maximum current except for circuits with a small number of inputs. In order to reduce the complexity, the current drawn by the logic gates is abstracted as a current

\*This work was performed while the authors were at University of Illinois, Urbana-Champaign

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waveform and the power distribution network is analyzed with that waveform. Using a lowerbound current waveform to estimate the maximum voltage drop in the power distribution network does not yield the maximum voltage drop values. All the above techniques simulate the power distribution network with R/RC/RLC models using macroblock current waveforms obtained heuristically or by simulation for a few vectors and are not guaranteed to generate the worst-case or even the best-case voltage variation at nodes in the power distribution network. Another approach is to estimate the maximum envelope current for all the macroblocks in the circuit [8]. The maximum envelope current is an upperbound on all possible current waveforms drawn by a macroblock. In [9], the maximum envelope current for the macroblocks is used as a periodic waveform to estimate the maximum voltage drop for RC power distribution networks. Although simulation of the RC power distribution network with the maximum envelope currents gives the maximum voltage drops, this is not true for any RLC power distribution network. The solution may still be a lowerbound on the maximum voltage drop. Without a good estimate of the maximum value of the voltage drop for RLC power distribution network and no understanding of the factors that affect the maximum voltage drop at specific nodes, the design/optimization of the power distribution network becomes hard and several ad hoc techniques are typically used in the design process. Several important design decisions such as placement of decoupling capacitors and off-chip power distribution network design require an estimate of the worst-case voltage variation and an excitation that causes the worst-case values. In this paper, we present a technique that is guaranteed to generate the maximum value of the voltage drop for the RLC power distribution network.

In the proposed technique, we use the maximum envelope current for each macroblock to determine the maximum current magnitude at a particular frequency. This is described in Section 3. We assume that the high-frequency content greater than clock frequency would find a low-impedance path to the macroblock decoupling capacitances and focus on the high-frequency content less than clock frequency. We use the bounds on the current magnitude at different frequencies and the Rayleigh's energy theorem [10] to formulate a nonlinear optimization problem for the maximum voltage drop at a node due to the current excitations at another node. This is described in Section 4. The solution to the nonlinear optimization problem is derived using Lagrange multipliers [11] and it yields the maximum voltage drop at a node due to all possible current excitations at another node. The maximum voltage drop at a node due to all the current excitations is obtained as the sum of the voltage drop due to each of the current excitations. This is described in Section 5. The complexity analysis and the significance of the proposed technique are presented in Section 6. In Section 7, we present the experimental results. In Section 8, we give the conclusions. In the next section, we formulate the problem.

## 2. PROBLEM FORMULATION

The number of nodes in the power distribution network can be extremely large because the power distribution network connects to every transistor in an integrated circuit. But a powerbus is typically designed as a hierarchical structure in which the top-level power-grid connects to the macroblocks and the power distribution network inside the mac-

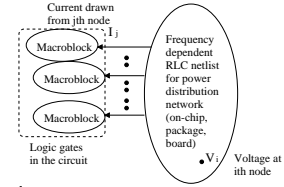


Figure 1: Hierarchical power bus analysis

robblock connects to the logic gates. All existing techniques [1, 2, 3, 4, 5, 6] and the proposed technique use the hierarchical abstraction with minor variations. Fig. 1 shows the schematic diagram of the proposed powerbus analysis using the macroblock current waveforms. In order to guarantee that the voltage drop in the power distribution network is maximum over all input vectors, the current waveform that is used to abstract the logic gate behavior should be valid over all the vectors. In this work, we use the maximum current envelope for the macroblocks which is an instant-wise upper bound over all possible current waveforms drawn by the macroblock in a clock-cycle [8]. In the remainder of this section, we give the expressions for the voltage drop at nodes in the power distribution network in terms of the current excitations in the frequency domain.

All voltages expressed in this paper are actually voltage deviations from the nominal value (reference value). This means that the same analysis is valid for both the power and ground busses. In the remainder of this paper, we present the analysis of the power (Vdd) bus. Furthermore, the maximum voltage variation from the reference value is referred to as the maximum voltage drop in this paper. We also assume that every node in the powerbus network has a capacitance to the reference node. The node voltages and currents in the frequency domain for a RLC power distribution network can be written as,

$$Y(f)V(f) = I(f) \quad (1)$$

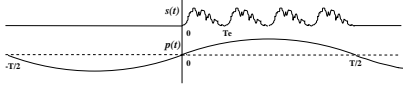
where,  $V(f)$  is a vector of all node voltages,  $I(f)$  is the current vector, and  $Y(f)$  is the complex admittance matrix. The admittance matrix has the capacitance terms on the diagonal and the resistance/ inductance terms on the diagonal and also as the off-diagonal terms of the matrix. The matrix  $Y(f)$  is complex, symmetric and diagonally dominant. Given the current excitation  $I(f)$  of a particular frequency, the voltage variations at that frequency can be obtained by computing the inverse of the  $Y(f)$  matrix and multiplying with  $I(f)$ . Let  $f_k$  be a frequency at which the voltage at node  $i$ ,  $V_i(f_k)$  is computed. The node voltage  $V_i(f_k)$  can be expressed as,

$$V_i(f_k) = Z_{i1}(f_k)I_1(f_k) + \dots + Z_{im}(f_k)I_m(f_k) \quad (2)$$

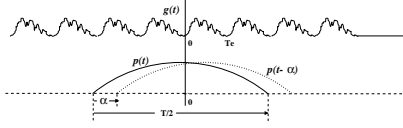
where,  $(Z_{i1}(f_k), \dots, Z_{im}(f_k))$  correspond to the elements of the  $i$ th row of  $Y^{-1}(f)$ . Let  $V_{ij}(f_k)$  denote the voltage drop at node  $i$  due to the current excitation at node  $j$  for a particular frequency  $f_k$ . The magnitude of voltage drop at node  $i$  due to the current excitation at node  $j$  for a particular frequency  $f_k$  can be written as,

$$|V_{ij}(f_k)| = |Z_{ij}(f_k)||I_j(f_k)| \quad (3)$$

Since  $|Z_{ij}(f_k)|$  is a constant, maximization of the magnitude of voltage drop  $|V_{ij}(f_k)|$ , requires the maximization of the current excitation  $|I_j(f_k)|$  for that frequency. In the next section, we present a technique to estimate the maximum value of the current magnitude at a particular frequency.



**Figure 2: Periodic waveform  $s(t)$  for maximum magnitude at a specified frequency**



**Figure 3: Maximizing the integral of the product over all time-shifts**

This is the maximum value of the current magnitude at a particular frequency over all possible time-domain current waveforms. These results are used in the subsequent section to formulate an optimization problem for the maximum voltage drop in the RLC power distribution network.

### 3. MAXIMUM CURRENT MAGNITUDE AT A PARTICULAR FREQUENCY

The time-period of the maximum envelope current is represented by  $T_e$ . If the maximum envelope current is repeated indefinitely, we get a periodic current waveform that is an instant-wise upper bound over all possible current waveforms drawn by a macroblock. We refer to this waveform as  $g(t)$ . If suppose we set the current waveform in alternate cycles of  $g(t)$  to zero, we get a periodic waveform in which the current is zero for duration  $T_e$ , followed by the maximum envelope current for duration  $T_e$ . We refer to the new periodic waveform as  $s(t)$  and it has a time-period  $2T_e$ . It is possible that the waveform  $s(t)$  gives the maximum magnitude of the current at frequency  $1/2T_e$ . Fig. 2 shows another waveform  $s(t)$  that may give the maximum magnitude of the current with a time-period  $8T_e$ . The goal is to reconstruct time-domain periodic waveforms ( $s(t)$ ) from  $g(t)$  for various frequencies such that the the magnitude of the current at that frequency is maximized. In addition, no other periodic or aperiodic waveform can generate a larger magnitude frequency component.

The problem of finding the maximum magnitude ( $C$ ) for a particular frequency based on the Fourier series requires the estimation of the maximum value of the integral of the product of the periodic maximum envelope current  $g(t)$  and the positive part of the sine/ cosine function at that frequency. This is graphically illustrated in Fig. 3. For arbitrary envelope currents, the value of the integral changes with time shifts. In order to obtain the maximum value for  $C$ , the following integral has to be maximized over all possible values of  $\alpha$ ,

$$R_\alpha = \int_{-\infty}^{\infty} g(t)p(t-\alpha)dt \quad (4)$$

PROPOSITION 1. *The maximum value of  $R_\alpha$  is given by,*

$$R_\alpha^{max} = g_0 P(0) + 2 \sum_{n=1}^{\infty} |g_n| |P(\frac{n}{T_e})| \quad (5)$$

where  $g_i$  denotes the Fourier series for  $g(t)$  and  $P(f)$  is the frequency domain representation of  $p(t)$ .

PROOF. Omitted due to lack of space  $\square$

Although the above expression contains an infinite summation, the values of  $|g_n|$  decrease rapidly with  $n$  and are insignificant for  $n > 20$  for any realistic  $g(t)$ . Hence, the summation can be approximated as a finite summation without a significant loss of accuracy. For an arbitrary maximum envelope current, Eqn. 5 can be used to compute the maximum value bound on  $C$  and it also corresponds to  $|I_j^{max}(f_k)|$ , the maximum current magnitude at a frequency.

### 4. MAXIMUM VOLTAGE VARIATION WITH RLC MODELS

The magnitude of voltage drop at node  $i$  due to the current excitation at node  $j$  for a particular frequency  $f_k$  can be computed using the  $|I_j^{max}(f_k)|$  value computed in the previous section. This process can be repeated for different frequencies. Given a set of frequencies  $S = \{f_1, \dots, f_k, \dots, f_n\}$  that could be excited by a hypothetical time-domain waveform, the voltage drop at node  $i$  due to the current excitation at node  $j$  for the set of frequencies  $S$  can be written as,

$$V_{ij}(S) = \sum_{k=1}^n V_{ij}(f_k) = \sum_{k=1}^n Z_{ij}(f_k) I_j(f_k) \quad (6)$$

The maximum magnitude of the voltage drop which is also the maximum time-domain voltage drop can be written as,

$$|V_{ij}^{max}(S)| = \max \sum_{k=1}^n |Z_{ij}(f_k)| |I_j^{max}(f_k)| x_k \quad (7)$$

where  $0 \leq x_k \leq 1$  and it denotes the fraction of the maximum current magnitude for frequency  $f_k$  that is included to compute the maximum magnitude of the voltage drop. The maximum magnitude of the voltage drop corresponds to the maximum time-domain voltage drop. A trivial upperbound can be obtained by setting all the values of  $x_k$  to 1. In reality, all the frequency components may never be maximum for any time-domain signal. Time-domain signals with the maximum magnitude at a particular frequency often have minimum magnitude at some of the other frequencies. In the remainder of this section, we describe the use of Rayleigh's energy theorem to obtain constraints on the variables  $x_k$ .

For an arbitrary signal  $u(t)$  and its Fourier transform  $U(f)$ , Rayleigh's energy theorem (Parseval's theorem) states that,

$$\int_{-\infty}^{+\infty} |U(f)|^2 df = \int_{-\infty}^{+\infty} |u(t)|^2 dt \quad (8)$$

This theorem states that energy measured in time and frequency domains is identical for a signal. It is valid for any periodic or aperiodic signal. Consider a current waveform  $i(t)$  that is obtained by duplicating the maximum current envelope for a time duration called the computation time ( $T_e$ ). The value of  $T_e$  is chosen to be an integral multiple of  $T_e$ . The current waveform  $i(t)$  is an aperiodic waveform of duration  $T_e$  and it is an upperbound over all possible waveforms drawn in that duration. Since  $i(t)$  is aperiodic, its Fourier transform  $I(f)$  would be a continuous waveform. In section 3, we presented a technique to compute the maximum current magnitude at a particular frequency  $f_k$ . The current magnitude at a particular frequency  $f_k$  can be represented as,

$$|I(f_k)| = |I_j^{max}(f_k)| x_k \quad (9)$$

The energy in the frequency domain over a set of frequencies  $S$  can then be written as,

$$\sum_{k=1}^n |I(f_k)|^2 = \sum_{k=1}^n |I^{max}(f_k)|^2 x_k^2 \quad (10)$$

Substituting this in the Rayleigh's energy theorem,

$$\sum_{k=1}^n |I^{max}(f_k)|^2 x_k^2 \leq \int_0^{T_c} |i(t)|^2 dt \quad (11)$$

The inequality comes from the fact that the set  $S$  does not include all the frequencies in the Fourier transform of  $i(t)$ . Using the linear objective function of the form  $\sum_{k=1}^n a_k x_k$  given by Eqn. 7 and the quadratic constraint of the form  $\sum_{k=1}^n w_k x_k^2 \leq c$  given by Eqn. 11, an optimization problem can be formulated for the maximum voltage drop. This nonlinear optimization problem is described in the next section.

The remaining issues are assigning values to the size of set  $S$  and the frequency components in set  $S$ . One of the components of set  $S$  is the DC component. It is computed from the aperiodic waveform  $i(t)$ . Apart from the DC component, sufficient number of frequencies must be selected in order to cover the peaks (local maxima) in the current magnitude and the impedance. One approach is to sample the frequencies uniformly. In general, it is not necessary to include all the frequencies because the optimization ensures that the peak voltage drop is captured. Note that the above formulation gives the solution to the maximum voltage drop at a node due to the current excitation. This has to be repeated for different current excitation to obtain the maximum voltage drop at a node due to each current excitation. The maximum voltage drop at a node due to all the current excitations is obtained as the sum of the voltage drop due to each of the current excitations. In the next section, we derive the solution to the nonlinear optimization problem using Lagrange multipliers.

## 5. SOLUTION WITH LAGRANGE MULTIPLIERS

The objective function given in Eqn. 7 can be written as,

$$\max \sum_{k=1}^n a_k x_k \quad (12)$$

where  $a_k > 0$ . The constraint Eqn. 11 can be written as,

$$\sum_{k=1}^n w_k x_k^2 \leq c \quad (13)$$

where  $w_k > 0$  and  $c > 0$ . The constraints on the bounds on the variables are represented as,  $x_k \leq 1 ; \forall k$ . The non-negativity constraint on  $x_k$  is not necessary because it is an inactive constraint. Using the remaining constraints we formulate the Lagrangian to transform the problem into an unconstrained optimization problem. In this formulation, scalars called the Lagrange multipliers are used for each of the constraints and this is added to the objective function. For this problem, the Lagrangian can be written as follows,

$$L(x, \lambda) = \sum_{i=1}^n a_i x_i + \lambda_0 [c - \sum_{i=1}^n w_i x_i^2] + \sum_{i=1}^n \lambda_i [1 - x_i] \quad (14)$$

The goal is to find the unconstrained maximum over  $x$  and the minimum over  $\lambda$  for  $L(x, \lambda)$  which produces the optimal

solution for the original problem. Suppose this solution say  $\tilde{x}, \tilde{\lambda}$  exists. Then the first-order conditions for a maximum of function  $L$  imply that the partial derivative with respect to  $x_i$  for all  $i$  must be 0. This means that,

$$\frac{\delta L(x, \lambda)}{\delta x_i} = a_i - \tilde{\lambda}_0 2w_i \tilde{x}_i - \tilde{\lambda}_i = 0 \quad \forall i \quad (15)$$

Rearranging,

$$\tilde{x}_i = \frac{a_i - \tilde{\lambda}_i}{2w_i \tilde{\lambda}_0} \quad \forall i \quad (16)$$

For inactive constraints the extreme (optimal) solution are achieved in the interior of the constraint. For these the  $\lambda_i$  value is 0. This in effect removes the inactive constraints from the Lagrangian. For active constraints the extreme (optimal) solution constraints are satisfied with an equality. For these constraints the first-order optimality conditions imply that the partial derivative with respect to  $\lambda_i$  must be 0. This is written as,

$$\frac{\delta L(x, \lambda)}{\delta \lambda_i} = 0 \quad (17)$$

If the constraint corresponding to  $\lambda_0$  is active, then  $\frac{\delta L(x, \lambda)}{\delta \lambda_0} = 0$ . This implies that,  $w_1 \tilde{x}_1^2 + \dots + w_n \tilde{x}_n^2 = c$ . Substituting the values of  $\tilde{x}_i$  from Eqn. 16 and rearranging terms,

$$\tilde{\lambda}_0 = \frac{1}{2\sqrt{c}} \sqrt{\sum_{i=1}^n \frac{(a_i - \tilde{\lambda}_i)^2}{w_i}} \quad (18)$$

If the constraint corresponding to any of the  $\tilde{\lambda}_i$  ( $i = 1, \dots, n$ ) is active, then  $\frac{\delta L(x, \lambda)}{\delta \lambda_i} = 0$  and it implies that,  $\tilde{x}_i = 1$ . The problem now is to determine which of the constraints are active and which are inactive. We solve this problem by first considering two extreme cases:

- Is the constraint corresponding to  $\tilde{\lambda}_0$  inactive and all the remaining constraints active?

The answer to the above question can be obtained by testing if all the  $\tilde{x}_i$  can be 1 and still satisfy the constraint corresponding to  $\tilde{\lambda}_0$  (Eqn. 13). This corresponds to checking if  $w_1 + \dots + w_n \leq c$ . If the above condition is true, then the constraint corresponding to  $\tilde{\lambda}_0$  is inactive and all the constraints corresponding to  $\tilde{\lambda}_i$  ( $i = 1, \dots, n$ ) are active for obtaining the maximum optimal solution. The optimal solution for the problem is obtained by setting all the  $\tilde{x}_i$  to 1 and the optimal value is  $\sum_{i=1}^n a_i$ .

- Is the constraint corresponding to  $\tilde{\lambda}_0$  active and all the remaining constraints inactive?

In order to answer the above question, we assume that the constraints corresponding to  $\tilde{\lambda}_i$  ( $i = 1, \dots, n$ ) are inactive and only the constraint corresponding to  $\tilde{\lambda}_0$  is active to compute the optimal solution analytically. From the solution, it is possible to determine the conditions for which the solution is valid and then present a recursive algorithm to compute the optimal value if the above conditions are not met.

Assuming that all the constraints corresponding to  $\tilde{\lambda}_i$  ( $i = 1, \dots, n$ ) are inactive implies that  $\tilde{\lambda}_i$  is zero for ( $i = 1, \dots, n$ ) and the constraint corresponding to  $\tilde{\lambda}_0$

is active. Substituting these values in Eqn. 18,  $\tilde{\lambda}_0$  is reduced to,

$$\tilde{\lambda}_0 = \frac{1}{2\sqrt{c}} \sqrt{\sum_{i=1}^n \frac{a_i^2}{w_i}} \quad (19)$$

and each  $\tilde{x}_i$  ( $i = 1, \dots, n$ ) from Eqn. 16 becomes,

$$\tilde{x}_i = \frac{a_i}{2w_i\tilde{\lambda}_0} \quad (20)$$

Substituting the value of  $\tilde{x}_i$  in the objective function,

$$a_1\tilde{x}_1 + \dots + a_n\tilde{x}_n = \frac{1}{2\tilde{\lambda}_0} \sum_{i=1}^n \frac{a_i^2}{w_i} \quad (21)$$

Substituting the value of  $\tilde{\lambda}_0$  in the above equation yields the optimal solution for these conditions,

$$\sqrt{c \sum_{i=1}^n \frac{a_i^2}{w_i}} \quad (22)$$

The above solution is the optimal value to the original problem only if none of the variables are larger than 1. This is because the constraints corresponding to  $\tilde{\lambda}_i$  ( $i = 1, \dots, n$ ) are assumed to be inactive. If the above condition is not true, then the optimal solution has to be computed by the techniques described below. Note that the optimal solution obtained by assuming that the variables are unbounded is an upperbound on the optimal solution for the problem with bounded variables.

If none of the above two extreme case hold true, then it implies that the constraint corresponding to  $\tilde{\lambda}_0$  is active and some of the remaining constraints are also active. We use the following recursive algorithm to determine the constraints corresponding to  $\tilde{\lambda}_i$  ( $i = 1, \dots, n$ ) that are active. Observe that if  $\frac{a_i}{w_i} > \frac{a_j}{w_j}$  then  $\tilde{x}_i \geq \tilde{x}_j$ , for any two variables that are not bounded. This is independent of the choice of the other variables that are bounded/ unbounded. Therefore we can use the following recursive algorithm to identify the variables that are set to 1. (inactive constraints)

1. Assuming all the remaining variables are unbounded, compute the optimal solution and identify the variable with the largest value.
2. If that variable is larger than 1, then set it to 1, update constraint Eqn. 13, drop the variable, repeat step 1.

The above algorithm terminates when the condition in step 2 is false. The worst-case complexity of the algorithm is linear-time in the number of variables. Once the algorithm terminates, the optimal solution for the original problem is computed as the sum of the optimal solution for the problem with reduced variables and the sum of the weights in the objective function corresponding to all previously dropped variables.

## 6. COMPLEXITY ANALYSIS

Let  $d$  denote the number of frequency components for which the numerical integration is performed in order to evaluate the Fourier series of the periodic maximum envelope current for a macroblock. Let  $m$  and  $l$  denote the

number of macroblocks and number of nodes in the circuit. The number of frequency components was previously denoted as  $n$ . The proposed technique has computational complexity in the following three steps: First, computation of the Fourier series for  $g(t)$  and the maximum current magnitude for all frequencies/ macroblock current waveforms:  $O(md + mn)$ . Second, computation of weights for objective functions:  $O(nl^3)$ . Third, solving the nonlinear optimization problem for the maximum voltage drop at all the nodes:  $O(nml)$ . The computational complexity of finding the maximum voltage drop at all the nodes in the RLC power distribution network is given by,  $O(md + mn + nl^3 + nml)$ . Clearly this is bounded by  $O(nl^3)$ , the cost of evaluating the inverse of the complex admittance matrix at different frequencies. Techniques to speed-up this computation can be used to reduce the complexity. As a comparison, the SPICE simulation for one current excitation with variable time steps could have a complexity greater than the above. Our technique gives the maximum over all excitations and a SPICE simulation for one current excitation would only give the voltage waveforms for that excitation. Since the analysis is performed in the frequency domain, frequency dependent resistance and inductance can be included in the analysis. Furthermore, the problem of setting the initial conditions in time-domain simulations for a trajectory that gives the maximum voltage variations does not appear in frequency-domain analysis. The maximum voltage drop computed by the proposed technique is an upperbound over all possible periodic, quasi-periodic, and aperiodic time-domain current excitation waveforms of the macroblocks. Since the power bus design techniques attempt to modify the impedance offered by the power distribution network to the current excitations at various frequencies, this technique can be used in the synthesis of robust power distribution networks.

## 7. EXPERIMENTAL RESULTS

Experimental results are presented for a 16-bit pipelined static CMOS adder using 0.35- $\mu\text{m}$ , 3-V technology. The RC interconnect parasitics of the power bus are extracted from the layout. Each resistance is replaced by two resistances with half the value and an inductance of a nominal value (0.05 nH) between the two resistances. A capacitance of nominal value (20 fF) is placed at each node without a capacitor to the reference node. The parameters of the bonding pin parasitics are chosen as  $L_p$  is 5 nH,  $R_p$  is 0.1  $\Omega$ , and  $C_p$  is 1 pF. The other end of the bonding pins is assumed to be connected to the reference voltage. The experimental results are generated for this RLC network.

In the first experiment, the maximum current envelope waveform for the macroblocks is used in the proposed technique and the SPICE simulation in order to evaluate the accuracy of the proposed technique. The computation time  $T_c$  is chosen to be 1000 times the time period of the current envelope  $T_e$ . Using these values, the maximum voltage drop in the RLC power distribution network is computed by the proposed technique described in the previous sections. SPICE simulation of the RLC power distribution network for different current excitations is performed to estimate the maximum voltage drop at nodes in the power distribution network. All quasi-periodic waveforms with a period of  $2nT_e$  (different integer  $n$  values) in which the maximum current envelope is applied for  $nT_e$ , followed by 0 current for  $nT_e$ , and repeated for the entire duration of the computa-

**Table 1: Worst-case voltage drops using the proposed technique and SPICE simulations**

Node	Proposed Technique (V)	SPICE sim. (V)
21	0.609	0.497
19	0.570	0.483
12	0.461	0.318
4	0.505	0.381
15	0.515	0.404
14	0.772	0.571
3	0.620	0.478
16	0.441	0.310
2	0.562	0.442
8	0.724	0.513
1	0.688	0.496
7	0.615	0.431
11	0.774	0.492
6	0.568	0.447
5	0.687	0.502
10	0.745	0.526

tion time, are used as the current excitations. In addition, SPICE simulations of the RLC power distribution network are performed for 2000 different random current excitations of 1000 cycles duration. For each of the random current excitations the current is initially assumed to be 0 for all the macroblocks. At each of the 1000 cycles, a unbiased coin is flipped and if the result is a 1 then the maximum envelope current for the macroblocks is used for that cycle. The maximum voltage drop at a node over all the SPICE simulation results is reported as the estimate of the maximum voltage drop at that node. Table 1 shows a comparison of the worst-case voltage drop at various nodes using the proposed technique and SPICE simulations. It can be seen that the proposed technique generates a tight upper-bound on the SPICE simulation results. The computation time for estimating the maximum voltage drop at all the nodes in the power distribution network using the proposed technique is 128.72 CPUs on a Sun UltraSPARC 5.

SPICE simulation of the adder circuit along with the power distribution network for a sequence of 100 random input vectors was also performed. These simulations are repeated 200 times for different input vector sequences to obtain the maximum voltage drop at nodes in the power distribution network. It was observed that the maximum voltage drop at nodes was under-estimated by a factor of 4 for some nodes when compared with the maximum voltage drop estimates obtained by the proposed method for a computation time of 100 cycles. In general, simulation of the circuit with random input vectors is not guaranteed to give good estimates of the maximum voltage drop except for circuits with a small number of inputs. The proposed technique uses the maximum envelope current of the macroblocks to generate tight upper-bound estimates of the maximum voltage drop with nominal computational resources. The mathematical analysis presented in this paper is for a general maximum envelope current. If certain assumptions can be made about the shape of the maximum envelope current, then more accurate expressions can be derived using the analysis presented in this paper. Also, a tight maximum envelope current for each of the macroblocks is required to reduce the pessimism in the voltage drop estimates.

## 8. CONCLUSIONS

In this paper, we presented a technique to estimate the worst-case voltage variation using a RLC model for the power distribution network. We use the maximum envelope current to estimate the maximum current magnitude at a particular frequency. An analytical expression is presented for the maximum value of the current magnitude at a particular frequency for any periodic/ aperiodic current waveform. We use the bounds on the current magnitude at different frequencies and the Rayleigh's energy theorem to formulate a nonlinear optimization problem for the maximum voltage drop at a node due to any current excitation at another node. The solution to the nonlinear optimization problem is derived using Lagrange multipliers and it yields the maximum voltage drop at a node due to any current excitation at another node. The maximum voltage drop at a node due to the current excitations at all nodes is obtained as the sum of the voltage drop due to each of the current excitations. Comparisons with SPICE simulations are presented to validate our approach. The CPU time requirements of the proposed technique are nominal. The significance of this work lies in its application to the design of robust power distribution network for circuits with high current demand.

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