Time-to-Failure Estimation for Batteries in Portable Electronic Systems

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ABSTRACT

Nonlinearity of the energy source behavior in portable systems needs to be modeled in order for the system software to make energy-conscious decisions. We describe an analytical battery model for predicting the battery time-to-failure under variable discharge conditions. Our model can be used to estimate the impact of various system load profiles on the energy source lifetime. The quality of our model is evaluated based on the simulation of a lithium-ion battery.

1. BACKGROUND

Electrical energy in portable systems is commonly supplied by *batteries*, or electrochemical cells, converting chemical energy into electrical energy. The time when the battery becomes fully discharged is the *lifetime* (time-to-failure) of the battery. This parameter is, perhaps, the most important: once the energy source is exhausted the system shuts down. The focus of this paper is on the development of an analytical battery model that can be incorporated into the system software for static and dynamic energy optimizations.

Modeling of batteries is difficult due to voltage and charge nonlinearities, which are especially pronounced when the discharge current varies with time. For the ideal power source the battery voltage $V(t)$ is constant over the entire discharge period, dropping from the open-circuit value V_{oc} to the cutoff value V_{cutoff} once the capacity is exhausted. A real battery, however, is exhausted earlier than the ideal one, and the voltage degrades with time. The time-to-failure L under the *constant* load I can be predicted based on empirical relationship due to Peukert [5]: $L = a/I^b$, where a and b are appropriate coefficients. This power-law relationship does not hold for the *variable* load (the current is changing over time).

There are several approaches to modeling the general discharge and charge behavior. The first approach is to solve (numerically) a set of partial differential equations that govern electrochemical processes which take place inside the battery [6]. An alternative much faster but less accurate approach is based on a simulation of a high-level equivalent representation of a battery: the reported equivalent representations were a PSPICE circuit [7], a VHDL model [2], and a Markov chain [8]. To the best of our knowledge, the Markov chain proposed in [8] is the most successful model in terms of its generality, accuracy, and practical value to a system designer. Such simulators can be successfully employed for system-level behavior characterization. This approach is adequate for an interactive design process; however, simulation speed is the key limiting factor. The next approach is to derive analytical expressions that relate load conditions and key battery parameters to the delivered energy [4, 9]. The authors of [4] consider special cases of the discharge process (such as diffusion-limited, reaction-limited, and ohmically limited cases) and obtain relationships between the discharge rate and the battery capacity. This work is of great importance to battery designers. Work [9] treats batteries from the user's point of view; the authors propose new design quality metric (discharge-delay product) and show that different load profiles may result in very different lifetimes. Analytical models inherently suffer from limited accuracy due to approximations. The last approach is based on purely statistical fit of experimental data: the authors of [10] used voltage-time measurements from several constant-rate discharge tests and fit the voltage-time curves into a Weibull failure model. The main disadvantage of this approach is that a statistical model is chosen ad-hoc and often is not robust due to the lack of its physical meaning.

Our effort is a combination of the analytical and statistical categories. We model the battery voltage degradation over time based on fundamental laws of chemical kinetics and mass transfer and estimate the unknown parameters statistically. Then, we show how to use our model to predict the state of the battery under a discharge rate varying with time.

2. MODEL DESCRIPTION

Let the battery voltage degradation be denoted by $\Delta V = V_{oc} - V$. A battery is considered fully discharged once $\Delta V = V_{oc} - V_{cutoff}$. Our goal is to construct an expression relating the voltage degradation to the discharge current i over time t (load profile). For a generic electrochemical cell, let the contribution of the electrical component be denoted by $\Delta V_E(i,t)$, and let the contribution of the chemical component be denoted by $\Delta V_C(i,t)$. Then,

$$\Delta V(i,t) = \Delta V_E(i,t) + \Delta V_C(i,t) \tag{1}$$

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The ΔV_E model is assumed to follow the Ohm's law: $\Delta V_E = ir$, where r is the value of the internal resistance. We derive the ΔV_C model based on general laws of reaction kinetics and mass transfer. The details of the model derivation and the key assumptions are presented in the Appendix. The final form is as follows:

$$\Delta V_C(i,t) = \frac{1}{\mu\alpha}\ ln\ \frac{i^2}{\beta_0 + \beta_1 i\sqrt{t} + \beta_2 i^2 t} \quad (2)$$

Now, we need to estimate the model parameters. The first step is to generate voltage-time data for a *fixed* load. We used the lithium cell simulator DUALFOIL [6] for a rechargeable lithium-ion battery. To cover sufficient range of currents we simulated ten constant loads resulting in the discharge times between 1 hour and 14 hours. The open-circuit voltage was $V_{oc} = 4.31V$, and the cutoff voltage was set to $V_{cutoff} = 2.80V$. The results are summarized in Table 1.

We approximated the times-to-failure in Table 1 by the Peukert's power-law model $L(I) = a/I^b$ (see Section 1) with the following estimated coefficients: $a = 2023$ and $b = 1.161$. For each of the ten voltage curves we estimated the β-coefficients of the ΔV_C model. The value of r was derived from the voltage drop at the beginning of the discharge for different fixed currents: $r = 0.0065$. We used $\alpha = 38.682$ and set $\mu = 0.06$. It can be seen that the value of the β-coefficients depend on the choice of the discharge rate. Physically, it means that the exchange currents and the limiting currents (see the Appendix) depend on the load. To capture this dependence at this point, we tabulate the β coefficients and, when needed, either interpolate or extrapolate the tabulated values for a given load.

We first tested our voltage model on the three constant current densities: 5, 10, and 15 A/m^2. The simulated and predicted voltage curves for the three currents are shown in Figure 1. The errors in predicting the time when the voltage reaches V_{cutoff} were 3% for 5 A/m^2, 1% for 10 A/m^2, and 10% for 15 A/m^2.

3. TIME-TO-FAILURE ESTIMATION

The basic approach is illustrated in Figure 2. Let the battery start discharging at $t = 0$ under the load I_1. At time t_1, the current is switched from I_1 to $I_2 < I_1$. The battery voltage just before the switch is $V(t_1) = V_1$, and the battery voltage just after the switch changes to $V_2 > V_1$. Next, we find time t_2 such that $V(t_2) = V_2$, assuming that the battery started discharging at time $t = 0$ under the load I_2. Then, the segment of the curve for I_2 starting at t_2 is simply shifted to the origin t_1, and the resulting time-to-failure is $L^* = L_2 - t_2 + t_1$. To simplify the discussion, we introduce two clocks - one shows the absolute (user) time t, and the other shows the relative (battery) time $\tilde{t} = t + \Delta t$, where $\Delta t = t_2 - t_1$. Note that after each switch the time shift changes.

Let $I(t)$ assume discrete values from the ordered set $S_I = (I_0, I_1, ..., I_k)$ over time t, where I_k is the *present* value of the discharge current. The user time of the current switch events are recorded, forming the ordered set $S_T = (T_0 = 0, T_1, ..., T_k)$. The battery voltage at time T_j, immediately *after* a switch from I_{j-1} to I_j, is denoted by V_j. The voltage values form the ordered set $S_V = (V_0, V_1, ..., V_k)$, where $V_0 = V_{oc} - I_0 r$. Let $\tilde{T}_j$ denote the relative (battery) time instance corresponding to the user time instance T_j. These battery time values form the ordered set $S_{\tilde{T}} = (\tilde{T}_0, \tilde{T}_1, ..., \tilde{T}_k)$. Then, the battery time is $\tilde{t} = t + \tilde{T}_j - T_j$. The battery voltage can now be evaluated for any present time t while the present current is constant. The time-to-failure is $L^* = T_{k+1}$ such that $V_{k+1} = V_{cutoff}$. In other words, if the current remains equal to I_k, the battery will become exhausted at time $t = L^*$.

Using our model, we can compute the voltage $V(I, \tilde{t})$ at any given time $\tilde{t}$ for a given current I. To find a time shift, we rely on the following equality, where $\tilde{t} = T_k + \tilde{T}_{k-1} - T_{k-1}$ (the battery time just before the switch):

$$\frac{\beta_{0,k-1} + \beta_{1,k-1} I_{k-1}\sqrt{\tilde{t}} + \beta_{2,k-1}(I_{k-1}\sqrt{\tilde{t}})^2}{\beta_{0,k-1}} = \frac{\beta_{0,k} + \beta_{1,k} I_k\sqrt{\tilde{T}_k} + \beta_{2,k}(I_k\sqrt{\tilde{T}_k})^2}{\beta_{0,k}} \quad (3)$$

In Equation 3, the β coefficients are from Table 1 for loads I_{k-1} and I_k. Equation 3 expresses the equality of the products of the concentration ratios $\frac{C_{O,c}}{C^*_{O,c}}$ and $\frac{C_{R,a}}{C^*_{R,a}}$ (see the Appendix) for load I_{k-1} at time $T_k + \tilde{T}_{k-1} - T_{k-1}$ and for load I_k at time $\tilde{T}_k$. Once the unknown $\tilde{T}_k$ is found, the value of V_k can easily be computed. Note that the difference between V_k and the battery voltage just before the switch is not as simple as the corresponding ohmic difference $(I_k - I_{k-1})r$.

Our approach is summarized in Figure 3. Sets S_I and S_T are the input; set S_V and the time-to-failure L^* are the output. The function $computeLifetimeForLoad(I)$ uses the Peukert's formula to find the lifetime for constant load I. The functions $computeVoltage(\cdot)$ and $computeTime(\cdot)$ are based on our model; for given input parameters these two functions output the battery voltage and the battery time, respectively.

4. EXPERIMENTAL RESULTS

We performed four tests with two periodic loads, one pulsed load, and one linear load. The current values for the periodic loads were set to 5, 10, and 15 A/m^2. The discharge duration for 5 A/m^2 was 6 min, for 10 A/m^2 - 3 min, and for 15 A/m^2 - 2 min, per period. The current values for the pulsed load were set to 20 (active) and 1 (idle) A/m^2. The discharge duration for 20 A/m^2 was 1 min; whereas, the duration for 1 A/m^2 was set to 4 min (20% duty cycle), per pulse. The linear load was generated as follows: the initial value was set to 1 A/m^2, and with every minute the load increased by 0.2 A/m^2.

Figure 4 shows the voltage curves for the 15-10-5 A/m^2 periodic current profile. Ten periods were applied and followed by a constant discharge at 10 A/m^2; the prediction error was 5%. Figure 5 shows the voltage curves for the 5-10-15-10 A/m^2 periodic current profile. Thirteen periods were applied before the battery became fully discharged; the prediction error was 3%. Figure 6 shows the voltage curves for the 20-1 A/m^2 pulsed load. After 50 periods the battery was discharged at 10 A/m^2 to the cutoff voltage; the prediction error was 20%. Figure 7 shows the voltage curves for the linear load. When the current value reached 20.4 A/m^2, the simulated battery voltage was at the cutoff level; the prediction error was 11%.

The model performed worst under the pulsed load due to the fact that it does not adequately handle the large amplitude and the number of transitions. For the linear load, where transitions were relatively smooth, the voltage predictions were more accurate.

5. CONCLUSION

We described the analytical model for the lithium-ion batteries. This model can be used to estimate the impact of various system loads on the energy source. The functional form of the model was derived based on physical principles; while, the model coefficients were fitted statistically, based on simulation results. The model predictions were compared with the simulation data under the constant discharge and the variable discharge conditions. In the former case the observed error was within 10%; and in the latter case the observed error was within 20%.

6. ACKNOWLEDGEMENTS

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7. APPENDIX

The electrode reaction - involving n electrons, oxidized species O, and reduced species R - can be represented as follows [1]:

$$O + ne^- \rightleftharpoons R \quad (4)$$

Let subscripts c and a denote the cathode and the anode, respectively. During discharge the battery voltage degrades from the open-circuit (equilibrium) value V_{oc} due to the ohmic drop ir, the cathode overpotential η_c, and the anode overpotential η_a [3]:

$$V = V_{oc} - ir + \eta_c - \eta_a \quad (5)$$

The following relationship describes reaction kinetics [1]:

$$\pm i = i_0 \left(\frac{C_O}{C_O^*} e^{-\frac{\mu n F \eta}{GT}} - \frac{C_R}{C_R^*} e^{\frac{(1-\mu) n F \eta}{GT}} \right) \quad (6)$$

In Formula 6, i_0 denotes the *exchange* current, C_O (C_R) denotes the *surface* concentration of oxidized (reduced) spieces, C_O^* (C_R^*) denotes the *bulk* concentration of oxidized (reduced) species, T is the temperature ($300K$), F is the Faraday's constant (96485.31 $C\ mol^{-1}$), G is the gas constant (8.3145 $J\ mol^{-1}\ K^{-1}$), and μ is the *symmetry* parameter. Before i, the plus sign is used for the cathode, and the minus sign is used for the anode.

On the right-hand side of the Formula 6 in parentheses, the positive term is related to the oxidation process, and the negative term is related to the reduction process. Next, we assume that oxidation is negligible at the cathode, and reduction is negligible at the anode. Then, one of the terms in the parentheses in Formula 6 can be dropped. After multiplying together the current expressions for the anode and the cathode, the following equation is derived:

$$i^2 = i_{0,c} i_{0,a} \frac{C_{O,c}}{C_{O,c}^*} \frac{C_{R,a}}{C_{R,a}^*} e^{\frac{(1-\mu_a) n F \eta_a}{GT} - \frac{\mu_c n F \eta_c}{GT}} \quad (7)$$

Let $\frac{nF}{GT} = \alpha$, and $\mu_c = 1 - \mu_a = \mu$. Note that $\eta_a - \eta_c = V_{oc} - V - iR = \Delta V_C$. Also, we may use the following expressions for the concentration ratios [1]:

$$\frac{C_{O,c}}{C_{O,c}^*} = 1 - \frac{i}{i_{lim,c}} = 1 - \frac{i\delta_c}{nFD_cA_cC_{O,c}^*} \quad (8)$$

$$\frac{C_{R,a}}{C_{R,a}^*} = 1 - \frac{i}{i_{lim,a}} = 1 - \frac{i\delta_a}{nFD_aA_aC_{R,a}^*} \quad (9)$$

In the above expressions, i_{lim} denotes the *limiting* current for the electrode, D denotes the diffusion coefficient, A denotes the area of the electrode, and δ denotes the diffusion layer thickness. Assuming that the concentration gradient is linear, the diffusion layer thickness can be approximated as $\delta = \sqrt{\pi D t}$, where t is time [3]. Then,

$$i^2 = [\beta_0 + \beta_1 i\sqrt{t} + \beta_2 (i\sqrt{t})^2] e^{\mu\alpha\Delta V_C} \quad (10)$$

$$\Delta V_C = \frac{1}{\mu\alpha}\ ln\ \frac{i^2}{\beta_0 + \beta_1 i\sqrt{t} + \beta_2 (i\sqrt{t})^2} \quad (11)$$

where:

$\beta_0 = i_{0,c} i_{0,a}$

$\beta_1 = -i_{0,c} i_{0,a} \left(\frac{\sqrt{\pi D_c}}{nFD_cA_cC_{O,c}^*} + \frac{\sqrt{\pi D_a}}{nFD_aA_aC_{R,a}^*} \right)$

$\beta_2 = i_{0,c} i_{0,a} \frac{\sqrt{\pi D_c}}{nFD_cA_cC_{O,c}^*} \frac{\sqrt{\pi D_a}}{nFD_aA_aC_{R,a}^*}$

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Load, A/m^2	Lifetime, min	β_0	β_1	β_2
2	841	3.8432	-0.0708	0.0001
3	557	8.6814	-0.1372	0.0003
4	415	15.4438	-0.2198	0.0004
6	271	34.6669	-0.4314	0.0009
7	229	47.0769	-0.5587	0.0012
8	197	61.3491	-0.7017	0.0015
12	116	136.5949	-1.4124	0.0031
13	101	159.7934	-1.6188	0.0035
14	89	185.8564	-1.9050	0.0045
16	68	240.8555	-2.3321	0.0047

Table 1: Simulated Lifetimes and Model Parameters.

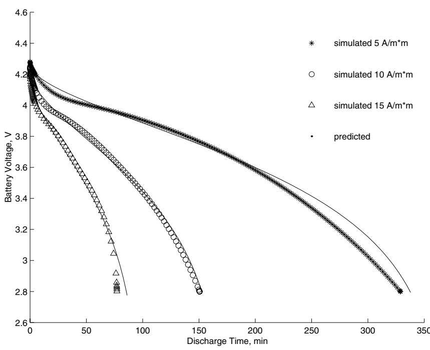

Figure 1: Three Predicted Voltage Curves.

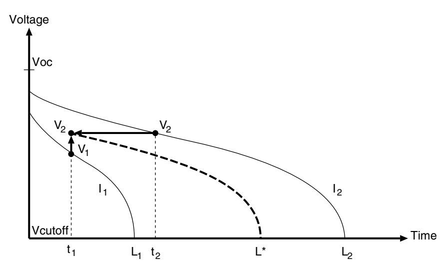

Figure 2: Voltage after Current Switch.

```
LifetimeEstimation (S_I, S_T)
 k = 0
 S_V[k] = V_oc − S_I[k] · r
 S_T̃[k] = 0
 while k < |S_T|
     k = k + 1
     L = computeLifetimeForLoad(S_I[k − 1])
     if S_T[k] − S_T[k − 1] < L − S_T̃[k − 1]
        t = S_T[k] + S_T̃[k − 1] − S_T[k − 1]
        S_T̃[k] = computeTime(t, S_I[k], S_I[k−1])
        S_V[k] = computeVoltage(S_I[k], S_T̃[k])
     else
        L* = S_T[k − 1] + L − S_T̃[k − 1]
        return L*, S_V
 L* = S_T[k] + L − S_T̃[k]
 return L*, S_V
```

Figure 3: Lifetime Estimation.

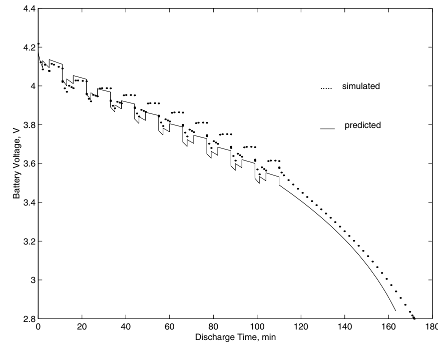

Figure 4: Voltage for 15-10-5 A/m^2 Periodic Load.

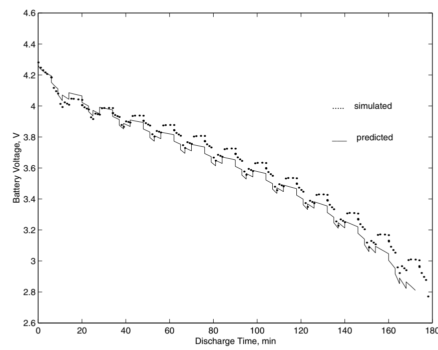

Figure 5: Voltage for 5-10-15-10 A/m^2 Periodic Load.

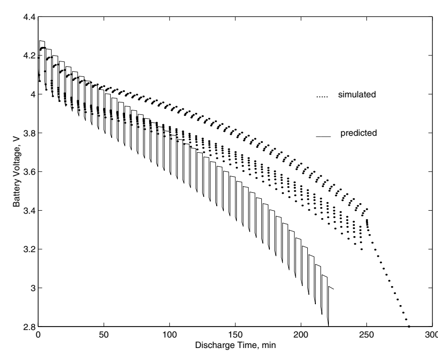

Figure 6: Voltage for 20-1 A/m^2 Pulsed Load.

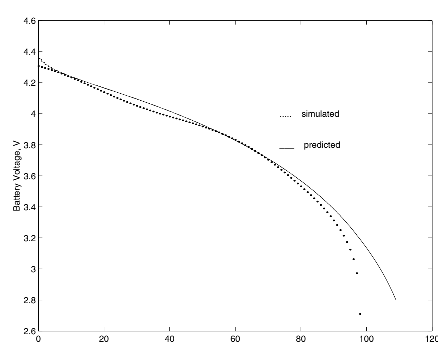

Figure 7: Voltage for 1 + 0.2/min A/m^2 Linear Load.