An Exact Analytical Time-Domain Model Of Distributed RC Interconnects for High Speed Nonlinear Circuit Applications *

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Abstract

Accurate simulation of interconnect effects is an increasingly critical step in high speed deep submicron design. With ever increasing frequency of digital/analog signals, the traditional lumped RC elements may not be accurate enough in modeling RC interconnects in VLSI applications due to the distributed nature of realistic interconnects. In this paper a novel analytic time-domain model for distributed RC interconnects is developed for application in nonlinear circuit simulators. The exact analytical solution is derived under the assumption of piecewise-linear signal waveforms at the two ports of the line. We have incorporated this model into a general-purpose circuit simulator using SWEC technique.

1 Introduction

The on-chip interconnects are generally modeled as lumped RC networks. But as the speed of ICs increases into GHz region, it is highly desirable to include the distributed nature of interconnects in order to ensure accurate simulation. The analysis of RC transmission lines is widely discussed in the literature. The traditional approach is to find the response of a finite RC transmission line with step input by utilizing some approximations [1] [2] [3]. In [1] and [4] the time-domain response at the far end of a RC transmission line with step input is obtained in a series expanded form, while in [5] [6] [7] the time-domain response for a finite-length open-ended RC line is derived by first solving the problem in the transform-domain, then inverting the resulting Laplace transform back into the time-domain. [6] also obtains the time-domain response for a finite RC transmission line by assuming a capacitive load, which is more realistic in VLSI applications. [8] gives the solution of the open-ended finite RC transmission line response by directly solving the diffusion equation in the time-domain as an infinite series. The direct solution method is also used in [9] to calculate the time-domain response of a finite distributed RC line with resistive source impedance and capacitive load impedance. This direct solution method utilizes the classical separation of variables method and gives the solution in the form of an infinite series [10] [11]. The associate boundary value problem presents great difficulties for the modeling of realistic VLSI interconnects when using direct method in the time-domain, as pointed by [11]. In order to model realistic VLSI interconnects with nonzero transient time for input signal, the finite ramp input signal model has been proposed in [12] to analyze RC interconnects under finite ramp input by assuming infinitely long transmission line. [13] treats a semi-infinite line by approximating the transfer function using a linear function. [14] further extends the work to compute the time-domain response for finite-length RC line under ramp input by summing distinct diffusions starting at either end of the line, and also presents a general recursive equation for calculating the high-order diffusion components due to reflections at the source and load ends.

Although these analytical methods for distributed RC line analysis are very useful in the characterization of RC interconnects, all of these algorithms cannot be easily incorporated into general-purpose circuit simulators because: (1) the nonlinear nature of the circuit needs to be considered, (2) the realistic input signals are not simple step or ramp signals, and (3) the crosstalk should be considered in modern VLSI design when several interconnects are packed closely together.

In this paper, we present a general exact method to characterize RC transmission lines directly in the time-domain which allows the interconnect model to be incorporated efficiently into nonlinear circuit simulation environment. The method utilizes the classical separation of variables approach to solving RC transmission line equation so that a closed-form representation of the transmission line as a two-port time-domain model is readily available. The method allows piecewise-linear signal representation and is not restricted to step or ramp signals as is currently the case. In our method, the RC interconnect is treated as a signle distributed line instead of an interconnection of lumped RC sec-

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tions; therefore, it is more accurate and reliable for high speed applications where the lumped model may not be accurate. The model has also been used to simulate crosstalk in coupled identical lines.

The algorithm has been incorporated into circuit simulator MIX [15], which is a mixed-mode simulation program. We will demonstrate by numerical examples that the algorithm is accurate and efficient.

The paper is organized as follows: in section 2 we derive an analytical solution of a distributed RC line under piecewise-linear signal model excitation. In section 3 we derive time-domain two-port model, and extend the method to identically coupled lines. Examples and numerical results are given in section 4. In section 5 we give conclusions.

2 Solution of RC Line Under Piecewise-Linear Signal Model Excitation

In many circuit simulators, the time-domain solution is carried out under a piecewise-linear approximation of the signal waveforms. An interconnect can be viewed as a black-box, and the relationship between the voltages and currents at the two ports of the black-box are utilized to construct the system matrix together with the nolinear devices in the circuit. Our approach is to solve the interconnect equations analytically in the time-domain during each time step in which the piecewise-linear approximation holds. Therefore the circuit system matrix originally derived from nonlinear devices can be quickly updated to take into account the effects of the interconnects. As a result, interconnects in the circuit can be included without much overhead and without applying reduced-order modeling.

Consider the initial boundary value problem for the diffusion equation that characterizes RC interconnects:

$$\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial V}{\partial t} \tag{1}$$

with the following boundary conditions:

$$\begin{aligned} V(t,0) &= V_1^{n-1} + \frac{V_1^n - V_1^{n-1}}{t_n - t_{n-1}}(t - t_{n-1}) \\ V(t,L) &= V_2^{n-1} + \frac{V_2^n - V_2^{n-1}}{t_n - t_{n-1}}(t - t_{n-1}) \end{aligned} \tag{2}$$

and initial condition:

$$V(t_{n-1}, x) = f(x) \tag{3}$$

where R,C are the distributed resistance and distributed capacitance, respectively. L is the length of the RC line. V_1^{n-1} denotes the voltage of the first port of the RC line at time t_{n-1}, and similarly V_2^n denotes the voltage of the second port of the RC line at time t_n, and so on. Note that we approximate the voltage waveforms at the two ports of the RC line by piecewise-linear functions.

The above equations include inhomogenous boundary conditions. The equations can be transformed into homogenous ones as follows. After denoting $V(t,0)$ as $V_1(t)$ and $V(t,L)$ as $V_2(t)$, we apply the transformation:

$$\begin{aligned} V(t,x) &= V_1(t) + (V_2(t) - V_1(t))\frac{x}{L} \\ &\quad + \beta_1 x(L-x) + \beta_2 x^2(L-x) + \phi(t,x) \end{aligned} \tag{4}$$

where

$$\begin{aligned} \beta_1 &= -\frac{(2V_1^n + V_2^n - 2V_1^{n-1} - V_2^{n-1})RC}{6(t_n - t_{n-1})} \\ \beta_2 &= \frac{(V_1^n - V_2^n - V_1^{n-1} + V_2^{n-1})RC}{6L(t_n - t_{n-1})} \end{aligned} \tag{5}$$

After the transformation, Eq(1) becomes

$$\frac{\partial^2 \phi}{\partial x^2} = RC\frac{\partial \phi}{\partial t} \tag{6}$$

with zero boundary conditions

$$\phi(t,0) = 0 \qquad \phi(t,L) = 0 \tag{7}$$

The initial conditions become

$$\begin{aligned} \phi(t_{n-1},x) &= f(x) - [V_1^{n-1} + (V_2^{n-1} - V_1^{n-1})\frac{x}{L} \\ &\quad + \beta_1 x(L-x) + \beta_2 x^2(L-x)] \end{aligned} \tag{8}$$

Eq(6) can now be easily solved by the method of separation of variables. The solution is:

$$\phi(t,x) = \sum_{n=1}^{\infty} B_n sin(\frac{n\pi}{L}x)e^{-\frac{n^2\pi^2}{L^2RC}t} \tag{9}$$

B_n can be determined from the initial conditions. After some mathematic deviations, we get

$$\begin{aligned} B_n = e^{\frac{n^2\pi^2}{L^2RC}t_{n-1}} [\, & f_n + \frac{2RCL^2}{\delta t_n n^3\pi^3}V_1^n - \frac{2RCL^2(-1)^n}{\delta t_n n^3\pi^3}V_2^n \\ & + \frac{2RCL^2((-1)^n V_2^{n-1} - V_1^{n-1})}{\delta t_n n^3\pi^3} \\ & + \frac{2(V_2^{n-1}(-1)^n - V_1^{n-1})}{\pi n}\,] \end{aligned} \tag{10}$$

where $\delta t_n = t_n - t_{n-1}$ and

$$f_n = \frac{2}{L}\int_0^L f(x) sin(\frac{n\pi}{L}x)\, dx \tag{11}$$

Finally, the solution of the diffusion equation(1) is:

$$\begin{aligned} V(t,x) &= V_1(t) + (V_2(t) - V_1(t))\frac{x}{L} \\ &\quad + \beta_1 x(L-x) + \beta_2 x^2(L-x) \\ &\quad + \sum_{n=1}^{\infty} B_n sin(\frac{n\pi}{L}x)e^{-\frac{n^2\pi^2}{L^2RC}t} \end{aligned} \tag{12}$$

3 Time-domain Two-port Model of Distributed RC Lines

The current along the line can be calculated as

$$I(t,x) = -\frac{1}{R}\frac{\partial V}{\partial x} \qquad (13)$$

Substituting (12) into (13), we obtain the current at port 1 (x=0) as

$$I(t_n, 0) = \alpha_{11} V_1^n + \alpha_{12} V_2^n + \gamma_1 \qquad (14)$$

The coefficients α_{11},α_{12}, and γ_1 can be calculated after some mathematical manipulations, and the results are

$$\begin{aligned}
\alpha_{11} &= \frac{1}{RL} + \frac{CL}{3\delta t_n} - \sum_{n=1}^{\infty} \frac{2CL}{n^2\pi^2\delta t_n} e^{-\frac{n^2\pi^2}{L^2 RC\delta t_n}} \\
\alpha_{12} &= -\frac{1}{RL} + \frac{CL}{6\delta t_n} + \sum_{n=1}^{\infty} \frac{2CL(-1)^n}{n^2\pi^2\delta t_n} e^{-\frac{n^2\pi^2}{L^2 RC\delta t_n}} \\
\gamma_1 &= -\frac{(2V_1^{n-1} + V_2^{n-1})CL}{6\delta t_n} \\
&+ \sum_{n=1}^{\infty} \frac{2V_1^{n-1} - 2(-1)^n V_2^{n-1}}{RL} e^{-\frac{n^2\pi^2}{L^2 RC\delta t_n}} \\
&+ \sum_{n=1}^{\infty} \frac{2CL}{\delta t_n n^2 \pi^2} \left(\left(V_1^{n-1} - (-1)^n V_2^{n-1} \right) \right. \\
&\left. - \frac{n\pi}{RL} f_n \right) e^{-\frac{n^2\pi^2}{L^2 RC\delta t_n}} \qquad (15)
\end{aligned}$$

The current at port 2 (x=L) can be obtained in a similar way.

The above equations give a time-domain two-port model for the RC line at $t = t_n$ from the previous values at $t = t_{n-1}$. This model can be readily incorporated into circuit simulator in which the integration is carrying out for nonlinear devices from time t_{n-1} to t_n.

For identically coupled lines, a decoupling scheme [17] can be used. After decoupling, the coupled lines become uncoupled and the model we derive in section 2 and 3 can be applied to each line directly.

4 Numerical Experiments

The above algorithm has been incorporated into MIX [15], a multipurpose simulation program. The circuit simulation in MIX utilizes SWEC [16] technique, and the implementation of this two-port model into the simulator is done by the standard stamp approach.

The circuit of the first example is represented in Fig. 1. The distributed capacitance is 2.5×10^{-10} F/m, distributed resistance is 15000 Ω/m, and the length of the line is 2000μm. The circuit has muti-fanins and muti-fanouts with RC lines connection as well as nolinear complex gates. Fig. 2 shows the voltage waveform at output node B as well as results from SPICE simulation. We note that the signal glitch is due to rising and falling signals at inputs of the NAND gate.

In another example shown in Fig. 3, we consider the crosstalk between two parallel RC lines driven by the inverters. The distributed coupling capacitance is 2.5×10^{-10} F/m and each RC line is terminated with a loading capacitance 1pF. The input of one inverter switches from high to low and the input of the other inverter remains high. Fig. 4 shows the voltage waveforms at the affected line output and shows about 0.35v crosstalk voltage. To verify the results, we used eight RC lumped sections to model RC line and added cross coupling capacitance between the RC lines (with lumped elements) in PSPICE simulation since PSPICE does not have builded-in modes to simulate crosstalk. The results from PSPICE are also shown in the fig. 4.

The CPU time for simulating C17 circuit is 0.16s for our method, 1.55s for PSPICE. The simulations were performed on a SPARC 20 machine.

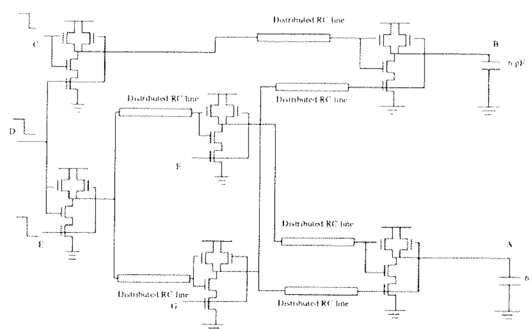

Figure 1: Schematic of The First Example

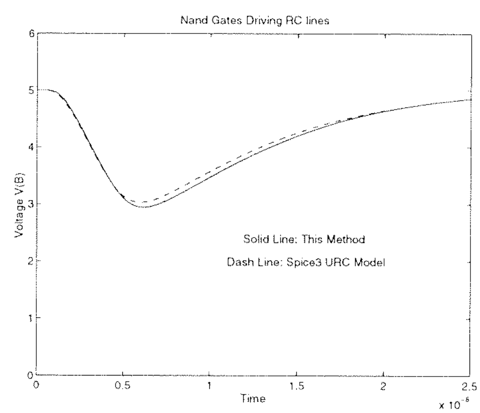

Figure 2: Simulated Waveform For The First Example

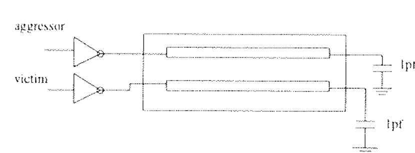

Figure 3: Schematic of Two Coupled Lines

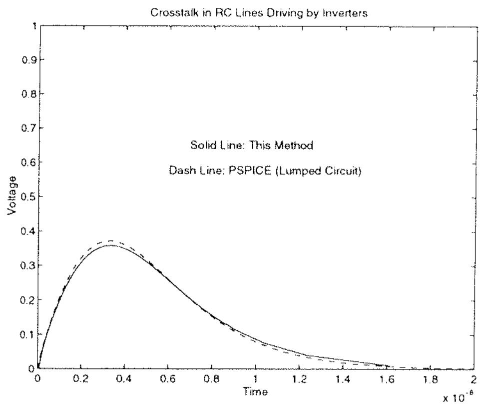

Figure 4: Crosstalk Voltage Waveform In the Victim Line

5 Conclusion

In this paper, we describe a novel technique to model distributed RC interconnects by analytically solving the diffusion equation in the time-domain. The resulting two-port model can be easily implemented into general-purpose circuit simulators, such as SWEC. The method also allows direct computation of crosstalk interference in identically coupled lines. Compare to other methods, this analytic technique can handle arbitrary signal waveforms in realistic high speed nonliner circuit environment. The analytic nature of the model as well as the time-domain formulation make this model efficient and accurate as shown in our experimental results. The extension of the method to the analysis on non-identical coupled lines is currently under investigation.

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