Theoretical Analysis of Word-Level Switching Activity in the Presence of Glitching and Correlation*

Janardhan H. Satyanarayana
Bell Laboratories
Lucent Technologies
101, Crawfords Corner Road
Holmdel, NJ 07733, USA.
E-mail: jana@lucent.com

Keshab K. Parhi
Department of ECE
University of Minnesota
200, Union Street S.E.
Minneapolis, MN 55455, USA.
E-mail: parhi@ece.umn.edu

Abstract

This paper presents a novel analytical approach to compute the switching activity in digital circuits at the word-level in the presence of glitching and correlation. The proposed approach makes use of signal statistics such as mean, variance, and autocorrelation. A novel expression is derived for the switching activity α_f at the output node f of an arbitrary circuit in terms of time-slot autocorrelation coefficient, the expected value, and the signal probability. The switching activity analysis of a signal at the word-level is computed by summing the activities of all the individual bits constituting the signal. A novel relationship between the correlation coefficient of the higher order bits of a normally distributed signal and the bit where the correlation begins is also presented. The proposed approach can estimate the switching activity in less than a second which is orders of magnitude faster than simulation based approaches. Simulation results show that the errors using the proposed approach are about 6% on an average and that the approach is well suited even for highly correlated speech and music signals.

1 Introduction

For well designed digital CMOS circuits, the dominant source of power consumption is due to the charging and discharging of the node capacitances and is computed as

$$P = \alpha.C_l.V_{dd}^2.f_{clk}, \quad (1)$$

where α (referred to as the switching activity) is the average number of $0 \rightarrow 1$ transitions in one clock-cycle, C_l is the load capacitance at the output node, V_{dd} is the power supply voltage, and f_{clk} is the clock frequency. It is clear that reduction in switching activity leads to reduction in power consumption. Therefore, methods to accurately estimate switching activity have been of interest to researchers in the recent past [1], . Most of these methods are non-theoretical in their approaches.

In this paper a novel approach for the word-level switching activity analysis of digital signal processing (DSP) systems in the presence of glitching and correlation is presented. In spite of the similarity with [3][6], the proposed approach differs from them in 1) taking into account the glitching effects 2) accurate computation of the break-points, 3) computation of the correlation coefficient, and 4) being accurate even for highly correlated speech and music signals. To the best of our knowledge, this is the first approach to include glitching in theoretical estimation of switching activity. The organization of this paper is as follows. Some preliminaries on the signal statistics and the computation of the switching activity are discussed in Section 2. A novel analytical approach for the analysis of word-level switching activity in the presence of glitching and correlation is presented in Section 3. This approach uses signal probabilities and correlation coefficients to compute the switching activity of the individual bits constituting a signal. In Section 4, new expressions for analytical estimation of the correlation coefficient are derived. It is shown that the proposed approach is well suited even for highly correlated signals. Experimental results to verify the switching activity analysis are presented in Section 5. Finally, the main conclusions of the paper are summarized in Section 6.

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2 Theoretical background

Consider a signal $x(n)$ which is L-bits wide. The signal is represented in two's complement notation as

$$x(n) = -x_L(n)2^{L-1} + \sum_{i=0}^{L-2} x_{i+1}(n)2^i \quad (2)$$

where $x_i(n) \in \{0, 1\}$ represents the i^{th} bit and n is the time index. According to this notation, $x_L(n)$ represents the most significant bit and $x_1(n)$ represents the least significant bit. Although the analysis presented in this paper is restricted to two's complement representation, the approach is general and holds true for any signal representation. Now, let us assume a normal distribution for the signal $x(n)$. Again there is no restriction on the distribution of $x(n)$. Normal distribution is assumed as a general case. Then, if the probability of the i^{th}-bit $i \in \{1, L\}$ being 1 is p_{x_i}, one can express the switching activity of the various bits of $x(n)$ as [6]

$$\alpha_{x_i} = p_{x_i}(1 - p_{x_i})(1 - \rho_{x_i}), \quad (3)$$

where ρ_{x_i} is the lag-1 temporal correlation of $x_i(n)$.

3 Word-level switching activity analysis in the presence of glitching and correlation

The switching activity at a node increases with increase in glitching activity. The glitching activity in turn depends on the various delays of the sub-circuits involved. Although it is difficult to exactly predict the exact glitching effects due to its complex dependence on the delays, a good approximation is still possible. In this paper, the notion of a time-slot [7] is employed to compute the switching activity in the presence of glitching.

Theorem 1 *Let us assume that every clock-cycle is divided into S time-slots numbered from 0 to $S-1$, where the duration of the time-slot is equal to the smallest gate delay of all the cells in the library. In this manner we can capture the glitching activity at the various output nodes in the circuit. Let us also assume that the inputs change only in the beginning of every clock-cycle. Moreover, let the output bit of the circuit at the beginning of a time-slot 'j' in every clock-cycle be defined as $f_j(Sn)$ where $j \in \{0, S-1\}$ and n is the time index. Then, the switching activity at the output node f in the presence of glitching is computed as*

$$\alpha_f = \sum_{j=0}^{S-1} \alpha_{f_{j,j+1}} = \sum_{i=0}^{S-1} p_{f_{j+1}} \left(1 - p_{f_j}\right) \left(1 - \rho_{f_{j,j+1}}\right) \quad (4)$$

where p_{f_x} is the probability of the output 'f' being '1' in time-slot x, and where

$$\rho_{f_{j,j+1}} = \frac{E\left[f_j(Sn) f_{j+1}(Sn)\right] - p_{f_j} p_{f_{j+1}}}{\sqrt{\left(p_{f_j} - p_{f_j}^2\right)\left(p_{f_{j+1}} - p_{f_{j+1}}^2\right)}}. \quad (5)$$

In the above equation the output of the circuit at time-slot 'S' is the same as the output of the circuit at time-slot '0' in the next clock-cycle.

Proof: Please refer to [8].

4 Estimation of the correlation coefficient

By performing various experiments it is observed that the correlation coefficient of the lower order bits is constant and equal to some value L_x, and the correlation coefficient of the higher order bits is constant and equal to some value U_x. For the intermediate bits, the correlation coefficient is found to decrease/increase in a linear manner from L_x to U_x. Therefore, it is proposed to divide the switching activity analysis into three regions: 1) lower, 2) middle, and 3) upper regions. The break-point P_0 separates the lower region from the middle region and the break-point P_1 separates the middle region from the upper region. Therefore, the correlation coefficient values for a signal $x(n)$ are defined as

$$\rho_{x_i} = \begin{cases} 0 & ; 1 \le i \le P_0 \\ \dfrac{(i - P_0)\,\rho_{x_c}}{P_1 - P_0} & ; (P_0 + 1) \le i \le (P_1 - 1) \\ \rho_{x_c} & ; P_1 \le i \le L. \end{cases} \quad (6)$$

The problem now reduces to computing the values of P_0, P_1, and ρ_{x_c} accurately.

A) Computation of P_0: Earlier works of [3][6] derived an expression which is suitable for uncorrelated signals. In this paper, an expression is derived for P_0 which is valid for both uncorrelated and correlated signals. If the correlation coefficient for the most significant bit of the input signal $v(n)$ is ρ_{v_c}, then the break-point P_0 is computed as

$$P_0 = \hookrightarrow log_2\left[2\sigma_v\left(1 - \rho_{v_c}\right)\right] \hookleftarrow \quad (7)$$

where $\hookrightarrow x \hookleftarrow$ is the nearest integer to x. If the output signal is of the form $x(n) = \sum_i h_i v(n-i)$, then the break-point P_0 is computed as the maximum of the break-points of the signals $\sum_i h_i v(n-i)$. The above expression has been tested for a variety of correlated and uncorrelated signals.

B) Computation of ρ_{x_c}: In this paper, a novel approach is presented for the computation of the correlation coefficient.

Theorem 2 *If the break-point of the lower region is P_0, then the correlation coefficient of the higher order bits of the output signal is given by*

$$\rho_{x_c} = erfc\left\{\frac{2^{P_0-1}-1}{\sigma_x\sqrt{2}}\right\} \quad (8)$$

where erfc(x) *stands for the complementary error function of x.*

Proof: Please see [8]. The above expression has been tested for a variety of correlated and uncorrelated signals.

C) Computation of P_1: In case of normal distribution, $x_{min} = \mu_x - 3\sigma_x$ and $x_{max} = \mu_x + 3\sigma_x$. Therefore, the dynamic range of the signal is $x_{max} - x_{min}$. Moreover if the correlation coefficient of the higher bits of the input signal is ρ_{v_c}, then the break-point P_1 is computed as

$$P_1 = \hookrightarrow log_2\left\{(x_{max} - x_{min})\left(\sqrt{1-\rho_{v_c}}\right)\right\} \hookleftarrow (9)$$

For normal distribution, this is approximated as

$$P_1 = \hookrightarrow log_2\left\{(6\sigma_x)\left(\sqrt{1-\rho_{v_c}}\right)\right\} \hookleftarrow . \quad (10)$$

This estimate is different from the ones proposed in [3][6] as those did not take the input correlation into account. The above relation has been verified using experiments on correlated and uncorrelated signals.

Therefore with the knowledge of P_0, P_1, and ρ_{x_c} the correlation coefficient ρ_{x_i} for all the bits can be computed using (6). The switching activity is finally computed using (3). In the next section several experiments are performed to verify the validity of the proposed approach.

5 Experimental results

In this section experimental results are presented for the word-level analysis of switching activity with and without glitching.

5.1 Word-level analysis of signals without glitching

In this section, the analytical expressions for the word-level switching activities are verified by performing various experiments. Two sets of experiments are performed; one where the word-length is varied and the other where the input signal is expressed as moving average, auto-regressive, etc. The experimental simulations are performed using MATLAB on a SUN SPARC-20 workstation.

A) Effect of signal representations: Experiments have been performed with AR, MA, and ARMA signals. Only the results of ARMA signals are presented in Table 1 for the sake of brevity. The results show that the average error using the proposed approach is about 6.9% for both uncorrelated and correlated signals as compared with 14.73% in [6].

B) Digital filter architectures: Consider the architecture of a direct form FIR filter with taps d_1=0.98, d_2=d_4=0.875, d_3=0.5, d_5=0.625. The filter is analyzed using the proposed approach and the results are presented in Table 2. It is clear from the results that the proposed approach is well suited for all kinds of signals. The average error using the proposed approach is about 5% as compared with 19.67% in [6].

5.2 Word-level analysis of signals with glitching

In order to consider the effect of glitching, two multiplier architectures namely the type-I and type-III multipliers [2][4] respectively, are analyzed. Again only the results of the type-I multiplier are presented in Table 3 for the sake of brevity. In the analysis it is assumed that the clock-cycle is divided into 4 time-slots. The

Table 3. Comparison of word-level switching activities for the type-I multiplier.

input signal	$\sum_{i=1}^{16}\alpha_{x_i} + \alpha_{x1_i} + \alpha_{x2_i} + \alpha_{x3_i}$		error (%)
	experimental	our approach	
v1(n)	34.76	33.76	-2.90
v2(n)	32.28	29.84	-7.53
v3(n)	10.94	10.11	-7.52
v4(n)	18.24	19.39	6.32
v5(n)	21.61	22.50	4.10

results show that the average error in the estimation of the switching activity is about 5.7%. It should be noted that in the computation of these switching activities the glitching effect has been taken into account.

6 Conclusions

A novel analytical approach is presented to compute the switching activity in digital circuits at the word-level in the presence of glitching and correlation. The

Table 1. Comparison of word-level switching activities for different approaches assuming that the input is a auto-regressive moving average type signal.

input signal	$\sum_{i=1}^{16} \alpha_{x_i}$			error (%)	
	experimental	[6]	our approach	[6]	our approach
v1(n)	3.42	3.07	3.30	-10.23	-3.46
v2(n)	3.07	2.83	3.07	-7.89	0.00
v3(n)	1.62	2.13	1.40	31.44	-13.77
v4(n)	2.30	2.51	2.34	9.04	1.69
v5(n)	2.78	2.54	2.32	-8.56	-16.48
v6(n)	2.63	2.80	2.55	6.13	-3.04
v7(n)	1.55	2.01	1.39	29.80	-10.10

Table 2. Comparison of word-level switching activities for different approaches for a 5-tap FIR filter.

input signal (x(n))	$\sum_{i=1}^{16} \alpha_{x_i}$			error (%)	
	experimental	[6]	our approach	[6]	our approach
x1(n)	51.80	49.14	48.17	-5.15	-7.02
x2(n)	46.71	43.87	43.97	-6.09	-5.88
x3(n)	18.84	26.75	18.72	42.03	-0.62
x4(n)	32.74	32.55	32.31	-0.60	-1.31
x5(n)	35.98	33.92	31.36	-5.70	-12.83
x6(n)	33.65	39.95	34.67	18.69	3.02
x7(n)	16.31	26.00	17.06	59.44	4.63

switching activity analysis of a signal at the word-level is computed by summing the activities of all the individual bits constituting the signal. A novel expression is also presented to compute the bit where the signal correlation begins. The proposed approach can estimate the switching activity in less than a second which is orders of magnitude faster than simulation based approaches. The average error for the word-level analysis is about 6% and the results show that the proposed approach is well suited for both uncorrelated and highly correlated signals.

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