Generalized Coupling as a Way to Improve the Convergence in Relaxation-Based Solvers

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Abstract

A new approach capable to improve the convergence of relaxation iterations is presented. The method is based on the use of generalized coupling patterns containing additional circuit elements. The parameters of such elements can freely be varied to control the convergence of iterations by suppressing local feedback in the decomposed circuit. In the case of time-domain circuit analysis by waveform relaxation method the adaptive strategy was formulated based on the dynamic estimation of input impedances of the parts in the decomposed circuit.

1. Introduction

Having been developed and first implemented in circuit simulators in the early seventies the relaxation techniques still remain very promising for large-scale circuit and system analysis. Until nowadays they have no alternative when for some reasons (such as excessive simulation time and memory requirements, incompatible model description in different parts, an intent of parallel computations) the simultaneous solution of the comprehensive system of equations is made prohibitive. In this case the algebraic or ODE system that describes a large circuit has to be decomposed into possibly many loosely coupled subsystems. The solutions to the subsystems are repeatedly recalculated by "guessing" or relaxing the behaviour of the surrounding subsystems until the unknown subvectors (or waveforms) converge [1]. Unfortunately, poor convergence properties often limit the applicability of relaxation algorithms. The problem arises when during the partitioning the circuit is cut across the signal flow paths and the resulting decomposed system cannot accurately reproduce the feedback existed in the original circuit.

The two types of feedback are often examined [2,3]. The firsts are global that run through feedback loops embracing several parts or functional blocks of the circuit. The seconds are local acting between neighbouring parts. As a rule, the signal flowing along the global feedback loop returns delayed. Therefore, the time-windowing technique [2,4] can effectively be exploited that divides the whole simulation interval into subintervals of such a length that the circuit couldn't "feel" the feedback. On the contrary, local feedback acts almost instantly and the above approach cannot improve the convergence in the same way. For this reason the relaxation-based simulators partition the circuit so that the nodes strongly connected by bilateral conductive elements should not be placed into different parts but have to be lumped together [4,3].

A number of partitioning methods have been proposed based on both the topological and quantitative information such as source-drain paths, the values and sizes of components in the circuit, etc. [5,6,4]. They all tried to fit a compromise between two opposites: if any tightly coupled nodes are not lumped together then relaxation algorithm will converge very slowly, but if too many nodes are lumped together, the advantages of using relaxation will be lost. The approach that lies outside this area is the overlapping partitioning [7], where additional nodes and common elements were included into both the subcircuits being separated. But this method increases the sizes of such parts and complicates the entire relaxation algorithm. In addition, it doesn't converge rapidly if bilateral conductive path exists through the parts separated.

In this paper we present a new approach to the convergence problem caused by local couplings in the decomposed system. It is based on the use of the generalized coupling pattern that suppresses the local feedback in the decomposed circuit. The method has demonstrated its effectiveness while simulating mixed system by its partitioning intotwo parts [8]. Here, we generalize this approach for the arbitrary number of parts and propose the adaptive method that combines the properties of different coupling patterns.

The paper is organized as follows. At first, we examine the basic coupling patterns (Section 2) and investigate their convergence factors. In Section 3 we propose a generalized coupling pattern and consider its properties. Further, in Section 4 we describe the corresponding matrix transformations for a linear system. In Section 5 we suggest an adaptive approach based on the generalized coupling pattern and consider its application to the nonlinear dynamic circuit analysis by waveform relaxation method. In Section 6 the method based on the automatic estimation of the controlling parameters is considered and the experimental results are present in Section 7.

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2. Basic coupling patterns

When the system of linear equations

$$AX+B=0,\ A\in R^{N\times N},\ detA\neq 0,\ B, X\in R^{N}, \quad (1)$$

is solved iteratively, the solution process can be described by the following expression [9]:

$$QX^{i+1}+SX^{i}+B=0 \text{ or } X^{i+1}=-Q^{-1}SX^{i}-Q^{-1}B=WX^{i}+G. \quad (2)$$

Here, X^i is an iterate of count i, $Q\in R^{N\times N}$ - nonsingular splitting matrix, $S=A-Q$ - a remainder, $W\in R^{N\times N}$ is an iteration matrix and G is a constant vector. The relaxation iterations (2) are known to converge if the spectral radius of the iteration matrix satisfies the condition:

$$\rho(W) = \rho[Q^{-1}(A-Q)] < 1. \quad (3)$$

The less is the radius, the faster is the convergence. While A represents the structure of the original system and it is not a subject to change, the splitting matrix strongly depends on the partitioning and the coupling between parts in the decomposed circuit. Below we'll show how the choice of a splitting scheme may affect the convergence factor.

Let us start the consideration with the circuit consisting of two parts ***A*** and ***B*** of input conductances y_a and y_b respectively connected by a conductance y_{ab}. x_1 and x_2 are the node voltages. Fig.1a,b represents two different coupling patterns which are used when the original circuit is decomposed across the conductance y_{ab}.

The first one (let us call it ***V***-type coupling) is used in all relaxation-based circuit simulators. It contains two voltage sources controlled by x_2^i and x_1^{i+1} and the conductive element y_{ab} presents in both its parts. It is essential to note that this coupling pattern produces a positive local feedback between x_1 and x_2. Thus, a positive increment Δx_1^{i+1} results in the positive increment Δx_2^{i+1} in the part ***B***, while the latter is conveyed at the next iteration to the first part and produces an additional positive increment Δx_1^{i+2}. The feedback factor is numerically equal to the predominant eigenvalue of the iteration matrix:

$$\sigma_V(W) = y_{ab}^2 / [(y_{ab}+y_a)(y_{ab}+y_b)] > 0. \quad (4)$$

As follows from (4), the error doesn't change its sign during iterating. The convergence is provided for any positive magnitudes of conductances. However, if $y_{ab} >> y_a,y_b$ then $\sigma_V(W) \approx 1$ and iterations converge slowly.

The second type of coupling (***I***-type coupling, see Fig.1b) is not very popular in the relaxation-based solvers since it doesn't secure the convergence. Primarily, it was used in [10] to couple the parts connected by pass transistors. The negative feedback occurs in the decomposed circuit since a positive increment Δx_1^{i+1} results in a positive increment ΔI^{i+1} of the current in the second part while the latter produces a negative increment Δx_1^{i+2} in the first part at the next iteration. Hence, the corresponding feedback factor is negative:

$$\sigma_i(W) = -(1/y_a)/(1/y_{ab}+1/y_b) < 0. \quad (5)$$

It follows from (5) that iterations converge rapidly if $y_a >> y_{ab}$ or $y_a >> y_b$, however, they diverge if the input conductance of the second part and y_{ab} in series exceeds the output conductance of the first part. Regardless to a rate of convergence, the resulting error changes its sign every iteration.

3. Generalized coupling pattern

As the comparison between ***V***- and ***I***- coupling patterns has demonstrated, their convergence properties contrast qualitatively. To accelerate the convergence, it would be desirable to build such a generalized coupling pattern whose feedback factor could occupy an intermediate position between (4) and (5).

Now, consider the coupling pattern shown in Fig.2 (which is called below ***IV***-coupling). Note, that the controlled sources of two different types are present in the first part of the equivalent circuit. There is also an additional element of conductance y^* placed in between the voltage and current sources.

First, we show that this pattern is consistent for any y^*. Actually, if $x^*=x_2^i=x_2$, the current through y^* is zero, hence, the solution obtained does satisfy the original circuit.

Further, considering Fig.2 one can derive a feedback factor, that is:

$$\sigma_{iv}(W) = y_{ab}^2(y^*-y_b)/[(y_{ab}y^*+y_ay^*+y_ay_{ab})(y_b+y_{ab})]. \quad (6)$$

If $y^*\rightarrow\infty$ then ***IV***-coupling pattern turns into that of ***V***-coupling since the current source in its right part has no effect, and $\sigma_{iv}\rightarrow\sigma_v$. On the contrary, if $y^*\rightarrow 0$, generalized pattern approximates to ***I***-coupling pattern and $\sigma_{iv}\rightarrow\sigma_i$. Evidently, there exists such a y^* that optimizes the convergence factor $\sigma_i<\sigma_{iv}<\sigma_v$. In fact, the optimum value is $y^*=y_b$ providing $\sigma_{iv}=0$. Under such a condition W becomes a nilpotent matrix of an index 2. Hence, two iterations are enough to reach the solution.

In summary, the generalized ***IV***-coupling pattern combines the properties of two basic couplings. Due to y^*, it has an additional degree of freedom. The magnitude of y^* does not alter the solution, which iterations converge to, however, varying y^* from zero up to infinity, one may control the convergence.

4. Generalized coupling in linear static case

Let us suppose that the circuit to be analyzed has been partitioned in accordance with some decomposition technique based on ***V***-type coupling. If (1) is a system of nodal equations, such a decomposition is known to produce block Gauss-Seidel iterations of the form:

$$Q^v X^{i+1} + S^v X^i + B = 0, \tag{7}$$

where Q^v is a (block) low triangular matrix and $S^v = A - Q^v$. Now, the question is, how the matrices in (7) have to be modified if ***V***-type coupling patterns in the decomposed circuit have been replaced by generalized ***IV***-patterns and (7) is transformed into:

$$Q^{iv} X^{i+1} + S^{iv} X^i + B = 0. \tag{8}$$

Let for some node k there exists a set $L_k=\{l: l>k \text{ and } a_{kl}\neq 0\}$. Then, while the correspondent equation (or subsystem) of (7) or (8) is being solved for x_k^{i+1}, the entries x_l, $l\in L_k$ are relaxed and the values x_l^i are used instead. If ***V***-type pattern coupling the nodes k and $l\in L_k$ is transformed into ***IV***-pattern with a conductance $y^*_{(k)l}$, the correspondent diagonal element of the splitting matrix should be decreased by $a_{kl}(y^*_{(k)l} - a_{kl})^{-1} a_{lk}$. Hence, the diagonal elements of Q^{iv} (S^{iv}) are expressed via those of Q^v (S^v) as:

$$q^{iv}_{kk} = q^v_{kk} - \sum_{l\in L_k} a_{kl}(y^*_{(k)l} - a_{kl})^{-1} a_{lk},$$

$$s^{iv}_{kk} = s^v_{kk} + \sum_{l\in L_k} a_{kl}(y^*_{(k)l} - a_{kl})^{-1} a_{lk}, \quad k=1...N. \tag{9}$$

All the rest entries in Q^{iv} and S^{iv} remain unaffected.

As a next step, the *block* ***IV***-coupling patterns could be proposed. Let $A_{kl}\in R^{N_k\times N_l}$ be the corresponding blocks in A formed due to the system decomposition. Then, the diagonal blocks of Q^{iv} and S^{iv} can as well be expressed as:

$$Q^{iv}_{kk} = Q^v_{kk} - \sum_{l\in L_k} A_{kl}(Y^*_{(k)l} - D_{(k)l})^{-1} A_{lk},$$

$$S^{iv}_{kk} = S^v_{kk} + \sum_{l\in L_k} A_{kl}(Y^*_{(k)l} - D_{(k)l})^{-1} A_{lk}. \tag{10}$$

Here $Y^*_{(k)l}\in R^{N_l\times N_l}$ is not any more a single conductive element but is thought as a convergence-controlling matrix describing the elements, placed into the block k being coupled with a block l, and $D_{(k)l}\in R^{N_l\times N_l}$ is a diagonal matrix of the entries: $d_{(k)l}(j,j)=\sum_{m=1}^{N_k} A_{kl}(m,j)$.

However, if the circuit to be analyzed has been partitioned into more than two parts and if each part may have connections to any other part, then, in general, the choice of the controlling matrices cannot improve the convergence unlimitedly. This is quite understandable; we cannot expect a diagonal matrix block modification be able to suppress an influence of all the nonzero entries of the remainder *S*. Nevertheless, ***IV***-coupling method considerably improves the convergence for such a general case as well.

It was found that ***IV***-modification acts in a way somewhat similar to that of successive overrelaxation (SOR) method; however, they differ in the following points:

- ***IV***-coupling results in the convergence of the rate not worse than that of either ***V***- or ***I***-type;
- The convergence-controlling conductances y^* have a clear physical meaning. Their magnitudes can merely be found by scrutinising the input conductances of adjacent parts in the decomposed circuit;
- For the most of circuit and system applications, the Jacobi matrix A is non-symmetric, the values of its entries may vary in many orders from step to step. This makes SOR method be impractical. On the contrary, as we'll demonstrate below, the modified***IV*** coupling is especially beneficial while analyzing strongly nonlinear circuits.

5. Generalized coupling for nonlinear dynamic circuit simulation

The presence of additional convergence-controlling elements y^* in the coupling patterns opens new possibilities in nonlinear dynamic circuit analysis. Not only the partitioning of one-way signal flow paths is made available, like that in combinational logic, but also the effective relaxation-based analysis of circuits where the direction of a signal flow may alternate. The partitioning across the pass-transistors, such as either in shift registers, multiplexers or in RAM cells in writing and reading mode, can easily be realized. Actually, at each segment of time the value and the type, resistive or capacitive, of the controlling conductance might be chosen so as to fit the input impedance of the neighbouring part.

Let us consider the MOS RAM cell operation (see fig.3). If the partitioning is made across the pass-transistor M12, the voltages $x1$ and $x2$ will be calculated in different parts. Three different modes of operation occur during the simulation interval: writing to the cell, idle, reading from the cell. Fig.4 shows the correspondent selection and write enable signals. Fig.6 illustrates the behaviour of first 4 waveform relaxation (WR) iterations for the different types of coupling.

In the ***writing mode,*** M12 is open and the write amplifier has a big output conductance, thus, $y_b = y_{amp} >> y_{ab}$, $y_{ab}=y_{M2} >> y_a=y_{cell}$. As follows from (4), $\sigma_v \approx 0$, therefore the ***V***-type iterations converge rapidly (see Fig.6a for the writing mode). On the contrary, ***I***-type iterations diverge and after the fourth iteration the error yields up to 150V, since $\sigma_i \approx -y_{ab}/y_a$, $|\sigma_i|>1$ (not shown in the picture).

In the ***idle mode*** M2 is closed, therefore $y_{ab} \approx 0$, $\sigma_v \approx \sigma_i \approx 0$, and both ***V***- and ***I***-type iterations converge rapidly.

In the ***reading mode***, however, $y_{ab} >> y_a, y_b$; $\sigma_v >> \sigma_i$, and the convergence is better for ***I***-type iterations. While ***V***-type iterations converge very slowly and nonuniformly, (see Fig.6a for the reading mode) the behaviour of ***I***-type iterations only depends on the ratio of the input capacitance of the sense amplifier to the output capacitance of the cell.

Thus, none of either ***V***- or ***I***-type coupling ensures an acceptable rate of convergence on the whole simulation interval. Therefore, the solution is obvious: one should apply the generalized ***IV***-coupling and control the parameter y^* in such a way that in the writing mode the coupling patterns behave like ***V***-type coupling while in the reading mode they should behave like that of ***I***-type. Fig.5 demonstrates the possible implementation of the convergence-controlling element for the given problem. Fig.6b illustrates the behaviour of ***IV***-iterations while $C^*=C_{amp}+C_{bus}=C_{opt}$. Only two WR iterations are needed to reach the desired solution. Moreover, the value C^* might be chosen with a big tolerance. If $C^*=0.5C_{opt}$ (see Fig.6c) or $C^*=2C_{opt}$ (see Fig.6d), the convergence is still fast enough, but the error may either alternate as for ***I***-type iterations or be monotonous as for ***V***-type ones.

6. Dynamic analysis based on the control element adjustment

The example considered above shows that the convergence of ***IV***-type iterations is very fast when the controlling parameters y^*, C^* are close to that of the input impedance of the neighbouring part. In general, however, with the exception of some special cases, we do not know these parameters a priori; and their analytical estimation might be very burdensome.

Fortunately, on the moment we are analysing the part ***A*** in the ***IV***-coupling pattern (Fig.3), we have already known the values (or waveforms in dynamic iterations) of x_2^i and I^i. This information can be used to estimate the parameters wanted.

Now, consider the case of WR dynamic iterations. Let the input impedance of the part ***B*** could be approximated by a conductance y^* and capacitance C^* connected in parallel. Then, if the simulation interval is split into several subintervals or time windows so that in each particular subinterval the functions $x_2^i - x_2^{i-1} = v(t)$ and $I^i - I^{i-1} = j(t)$ are almost linear,

$$v(t)=a_v+b_v t\,, \qquad j(t)=a_j+b_j t, \tag{11}$$

the parameters may be found from the equation

$$j(t)=y^* v(t)+C^* v'(t). \tag{12}$$

Substituting (11) into (12), we can express the desired parameters as:

$$y^*=b_j/b_v, \qquad C^*=(a_j b_v - a_v b_j)/b_v^2\,. \tag{13}$$

The expressions (13) are the key point to the control element adjustment. The process starts with $y^*=\infty$, $C^*=0$ (***V***-type coupling). After the first iteration performed within the same time window, the values y^*, C^* are taken from (13). To avoid accidental parameter dispersion only positive values of y^*, C^* are accepted, otherwise, the ***V***-type coupling is used. In a new time window the previous estimations y^*, C^* are used for the first iteration. Naturally, it is not necessary to store all the waveforms x_2^i, x_2^{i-1}, I^i, I^{i-1}: as soon as x_2^{i+1}, I^{i+1} have been found, the coefficients a_v, b_v, a_j, b_j are extracted and all the rest waveforms are no longer needed.

7. Experimental results

To illustrate the above approach, a line of 256 MOS cells was simulated attached to one bit-line of a RAM circuit. A bit sequence of hexadecimal codes 20-3F was first written into the addresses 00-1F and then read from the memory in the reverse order. Pspice 4.03 simulator was used with a specially elaborated program envelope to exchange waveforms between the parts, extract the controlling parameters, estimate pure solution time and stop the iterations. The runtimes for Cx486DX2/50 (the loading Pspice and excessive compilation are not included) are shown in Table 1. The switch model shown in Fig.5 and the adjustable model were used as a convergence-controlling element in the ***IV***-coupling patterns.

Table 1. Runtimes via # of parts, sec

# of parts	1	8	16
V-coupling	1610*	8130	7820
IV-coupling (switch model)	1610*	795	687
IV-coupling (adjustable model)	1610*	1176	1088

* Direct simulation, no partitioning. Since the program did not allow simultaneous solution for 256 cells, this estimation was obtained by the extrapolation on 32, 64 and 128 cells.

As Table 1 indicates, ***IV***-coupling approach makes the considerable reduction of the simulation time with respect to ***V***-coupling and the direct solution. The adjustable model enables more general approach since it doesn't use any preliminary information about the circuit but, of course, it invokes some extra operations to find the controlling parameters.

8. Conclusion

In this paper we introduced a new approach to the convergence problem of relaxation iterations based on the use of generalized ***IV***-coupling pattern. The two types of the convergence-controlling elements were proposed, switch-model elements and adjustable parameter elements. The experiments demonstrated the high effectiveness of the approach while simulating the circuits cut across the tight links. Furthermore, we can expect the performance of the proposed technique to be higher for larger circuits with respect to both direct simulation and ***V***-type iteration method.

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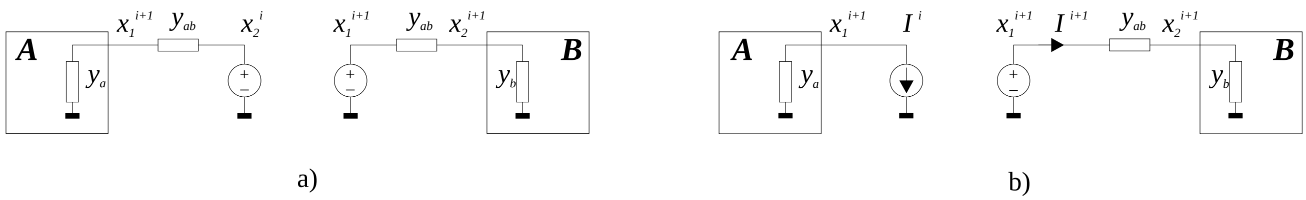

Fig.1 Basic types of coupling: V-type (a) and I-type (b)

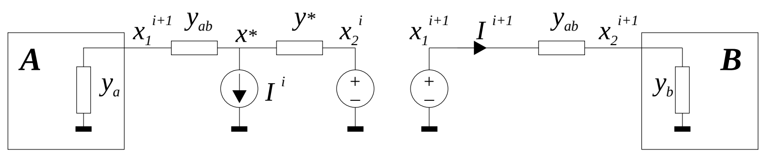

Fig.2 Generalized (IV) coupling pattern

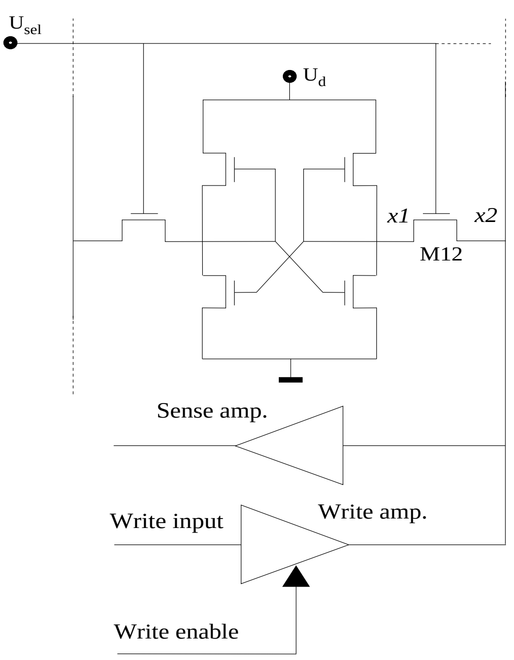

Fig.3 MOS RAM cell

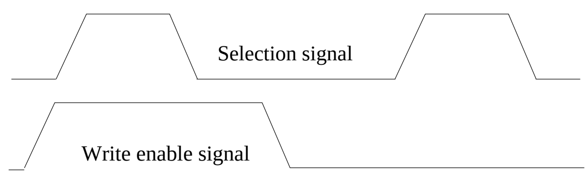

Fig.4 Controlling signals

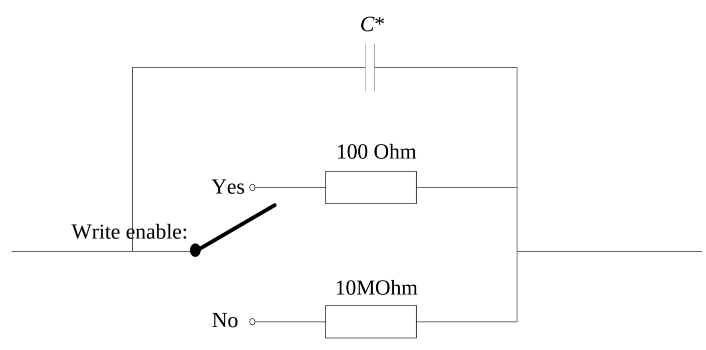

Fig.5 Convergence-controlling element for IV-coupling pattern

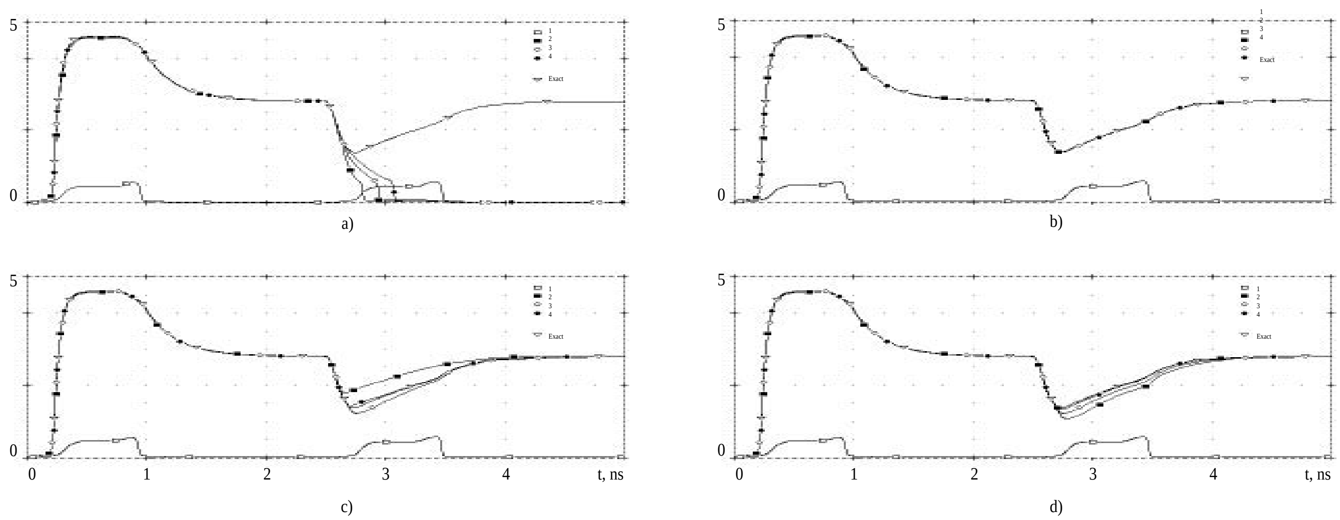

Fig.6 First WR iterations for (a) ***V***-type coupling; (b-d) ***IV***-coupling with $C^{*}=C_{opt}$ (b), $C^{*}=0.5C_{opt}$ (c) and $C^{*}=2C_{opt}$ (d)