Capturing Time-of-Flight Delay for Transient Analysis Based on Scattering Parameter Macromodel

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Abstract

The delay associated with transmission line networks consists of the exponentially charging time and a pure propagation delay. This propagation delay, so called timeof-flight delay, is particularly evident in long lines. When the time-of-flight is comparable to the input rise-time, it is difficult to capture the time-of-flight with a finite sum of exponentials. Therefore the time-of-flight must be captured explicitly from the transfer function of the circuit. In this paper, we give a precise definition of the time-of-flight together with some basic properties, and present an efficient method to capture the time-of-flight for general interconnect networks. Based on our scattering parameter macromodel, we can easily capture the time-of-flight during the network reduction while using lower order model to evaluate the charging delay. By capturing the time-of-flight delay, the accuracy of system responses can be greatly improved without significantly increasing computing time.

1. Introduction

Recently, an nth order extension of Elmore delay model [7] based on Padé approximation, such as Asymptotic Waveform Evaluation (AWE) [14], has been developed to approximate a higher order linear network using the waveforms generated by its lower order moments. However, the delay associated with transmission line networks consists of the exponentially charging time and a pure propagation delay representing the finite propagating speed of electromagnetic signals in the dielectric medium. This propagation delay, so called time-of-flight delay, denoted by τ_f , is particularly evident in long lines (See Fig. 1). As the time-of-flight of the signal across the interconnect is greater than, or comparable to, the input signal rise-time (i.e. long interconnects), it is difficult to capture the time-of-flight delay with a finite sum of exponentials [2, 13, 15] or an exponentially decayed polynomial function [5, 10]. The captured time-of-flight is also used as the lower bound for the delay of the lossy transmission lines [15].





Hence, the time-of-flight τ_f , more precisely the factor $e^{-s\tau_f}$, must be captured explicitly from the transfer function of the circuit. As we know, a transfer function will be called ideal if it is of the form $H(s) = e^{-s\tau_f}$. For $s = j\omega$, the magnitude identically equals to one, and the angle is proportional to the angle frequency ω . According to the shifting theorem of Laplace transform, if this ideal network is excited by a signal e(t), the corresponding response of the network will be $e(t - \tau_f)$. The response is the same as the excitation except that it is delayed in time by an amount of τ_f . That is, the response is equal to zero for $t < \tau_f$. Therefore, the time-of-flight is defined as the maximum delay during which the output response is zero for any finite input. $e^{-s\tau_f}$ is the corresponding factor in the frequency domain.

Several attempts have been made to capture the timeof-flight delay. They either require an explicit analytical expression of the transfer function which is impractical in general [4] or can only deal with one set of transmission lines [13, 3, 9]. Although the method proposed by paper [9] may be extended to general interconnect networks, it may not retain the passivity property of the original transmission line since the effective time-of-flight is approximated based on an approximated phase constant of the equivalent lumped circuit.

In this paper, we present a new method to compute the time-of-flight for arbitrary interconnect systems, not limited to one transmission line. The method is based on a scattering parameter macromodel [10]. The accuracy of

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output responses, due to the capturing of the time-of-flight, is greatly improved. In the next section, a precise description of the time-of-flight together with some basic properties are given. The scattering parameters of basic components and their time-of flight are described in Section 3. The network reduction process and how to keep track of the time-of-flight during network reduction are described in Section 4. Experiment results are given in Section 5 followed by conclusions.

2. Properties of Time-of-Flight

Recall that the time-of-flight is the maximum delay during which the output response is zero for a finite input signal. The computation can be approached from the properties of transfer function in the frequency domain. The following theorem precisely describes the time-of-flight.

Theorem 1: For any $\varepsilon > 0$, there exists positive constant s_0 , such that the time-of-flight τ_f of the transfer function H(s) satisfies the following

$$e^{-\varepsilon s} < |H(s)| e^{\tau_f s} < e^{\varepsilon s} \text{ for all } s \ge s_0$$
(1)

See [12] for the proof. Notice that the most commonly used definition of time-of-flight [1, 4]

$$\tau_f = \lim_{s \to \infty} -\frac{1}{2} \frac{d}{ds} \left[ln \frac{H(s)}{H(-s)} \right]$$
(2)

may be incorrect for some cases. For example, the transfer function of the transmission line circuit shown in Fig. 2 is

$$H(s) = \frac{2ZZ_c}{(Z + Z_c)^2 e^{\gamma} - (Z - Z_c)^2 e^{-\gamma}}$$
(3)

where $Z_c = \sqrt{(R + sL)/(sC)}$ and $\gamma = \sqrt{(R + sL)sCl}$. Obviously, the time-of-flight is \sqrt{LCl} . But according to (2), it is zero.



Figure 2. Transmission line circuit.

Since it is impractical to get an explicit expression of the transfer function in general, we may not be able to use Theorem 1 directly to capture the time-of-flight. The following corollaries of Theorem 1 will be used later to keep track of the time-of-flight during the proposed network reduction process.

Corollary 1: If the time-of-flights of the non-zero functions $F_1(s)$ and $F_2(s)$ are $\text{TOF}(F_1(s))$, and $\text{TOF}(F_2(s))$

respectively, then the time-of-flight of $F_1(s) \pm F_2(s)$ is $\text{TOF}(F_1(s) \pm F_2(s)) = \min(\text{TOF}(F_1(s)), \text{TOF}(F_2(s)))$ (4)

Corollary 2: If the time-of-flights of the non-zero functions $F_1(s)$ and $F_2(s)$ are $\text{TOF}(F_1(s))$, and $\text{TOF}(F_2(s))$ respectively, then the time-of-flight of $F_1(s)F_2(s)$ is

$$TOF(F_1(s)F_2(s)) = TOF(F_1(s)) + TOF(F_2(s))$$
(5)

Corollary 3: If the time-of-flights of the non-zero functions $F_1(s)$ and $F_2(s)$ are $\text{TOF}(F_1(s))$, and $\text{TOF}(F_2(s))$ respectively, then the time-of-flight of $F_1(s)/F_2(s)$ is

$$\operatorname{TOF}(F_1(s)/F_2(s)) = \operatorname{TOF}(F_1(s)) - \operatorname{TOF}(F_2(s))$$
(6)

Later we will show that these corollaries have clear physical meaning during the network reduction. Let us first describe the time-of-flight of scattering parameters of basic circuit components.

3. Time-of-Flight of Scattering Parameters of Basic Circuit Components

We use scattering parameters (S-parameters) to describe the components of interconnect systems. Scattering parameters is a powerful way to describe and model interconnects. They can be measured directly at high frequencies and they exists for all distributed-lumped circuit elements including open and short circuits. It can also describe transmission lines which is important in today's high-speed designs. A scattering matrix is employed to relate outgoing waves to incoming waves of a multiport [6]. For an n port component, the scattering matrix of the component can be defined as

$$S_{ji} = \frac{b_j}{a_i}\Big|_{a_k = 0, \, k \neq i} \qquad i, j = 1, 2, ..., n$$
(7)

where a_i is the incoming voltage wave at port *i* and b_j the outgoing wave at port *j*. Let V_i and I_i be the voltage and the current at port *i*. The wave parameters a_i and b_j relate to the circuit parameters as follows:

$$a_i + b_i = V_i$$
 and $a_i - b_i = Z_0 I_i$ (8)

where Z_0 is the reference impedance. The (7) and (8) can be used to derive scattering parameters of some basic components.

The components utilized to characterize a general interconnect network can be classified into four types [10]: 1) one-port impedance, 2) two-port impedance, 3) multiport interconnect node and 4) lossy transmission line (See Fig. 3).

The first three components (shown in Fig. 3(a-c)) are lumped components, i.e., electromagnetic waves propagate across the component virtually instantaneously. Therefore, the time-of-flights of S-parameters of these components are all equal to zero.



(c) Multiport interconnect node (d) Lossy transmission line **Figure 3. Four basic components.**

For an RLC transmission line shown in Fig. 3(d), the S-matrix is [6]

$$S = \frac{1}{2Z_0 Z_c \cosh(\gamma) + (Z_c^2 + Z_0^2) \sinh(\gamma)} \times \begin{bmatrix} (Z_c^2 - Z_0^2) \sinh(\gamma) & 2Z_c Z_0 \\ 2Z_c Z_0 & (Z_c^2 - Z_0^2) \sinh(\gamma) \end{bmatrix}$$
(9)

where $Z_c = \sqrt{(R + sL) / (sC)}$ is the characteristic impedance, $\gamma = \sqrt{(R + sL) sCl}$ the wave propagation constant and *l* the length of the line. Applying Theorem 1 to the S-matrix of the lossy transmission line, we find that the time-of-flights of both S_{11} and S_{22} are zero, but the time-of-flights of S_{12} and S_{21} are $\tau_f = \sqrt{LCl}$.

4. Keeping Track of Time-of-Flight during Network Reduction

Given the scattering parameters of individual components, we have described a systematic reduction algorithm [10] to reduce a distributed-lumped linear network to a multiport together with sources and loads of interest, as shown in Fig. 4. We can reduce a general net-



Figure 4. S-parameter based macromodel.

work based on two basic rules: Adjoined Merging Rule and Self Merging Rule. For scattering parameters of basic components, we can find the corresponding time-of-flight according to the Theorem 1 as described in Section 3. During network reduction, there are only four fundamental operations: addition, subtraction, multiplication and division. The three corollaries we derived in Section 2 are applied to keep the track of the time-of-flight.

Adjoined Merging Rule: Let X and Y be two adjacent multiports, with m ports and n ports respectively. Assume port k of X is connected to port l of Y, as shown in Fig. 5.



After merging X and Y, the resultant (n + m - 2) port has the following S-parameters:

$$S_{ji} = \begin{cases} S_{ji}^{(X)} + \frac{S_{ki}^{(X)} S_{ll}^{(Y)} S_{jk}^{(X)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} & i, j \in X \\ S_{ji}^{(Y)} + \frac{S_{li}^{(Y)} S_{kk}^{(X)} S_{ll}^{(Y)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} & i, j \in Y \\ \frac{S_{ki}^{(X)} S_{jl}^{(Y)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} & i \in X, j \in Y \end{cases}$$
(10)

Let us take a close look at the case where both port iand port j belong to the same component, say X, then S_{ii} is

$$S_{ji} = S_{ji}^{(X)} + \frac{S_{ki}^{(X)} S_{ll}^{(Y)} S_{jk}^{(X)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} \qquad i, j \in X$$
(11)

 S_{ji} consists of two terms, which implies that there are two paths for electromagnetic wave to propagate from port *i* to port *j*. The first term of (11) represents the first path on which the wave directly propagates from port *i* to port *j*. The second term represents the second path on which the wave propagates from port *i* to port *j* via port *k* (See Fig. 6). Obviously, the time-of-flight of the S_{ij} is the mini-



Figure 6. For $i, j \in X$, there are two paths for wave to propagate from port *i* to port *j*.

mal of the time-of-flight of these two paths, since the timeof-flight is the minimum time at which the output has nonzero response. This is the physical meaning of the Corollary 1. The time-of-flight of the second path is the sum of the time-of-flight of $S_{ki}^{(X)}$ and $S_{jk}^{(X)}$, since the path consists of two sub-paths. This is the physical meaning of the Corollary 2. Therefore, the time-of-flight of S_{ji} is

$$TOF(S_{ji}) = min(TOF(S_{ji}^{(X)}), TOF(S_{ki}^{(X)}) + TOF(S_{jk}^{(X)}))$$
(12)

Similarly, if port i belongs to X and port j belongs to Y, then

$$S_{ji} = \frac{S_{ki}^{(X)} S_{jl}^{(Y)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} \qquad i \in X, j \in Y$$
(13)

There is only one path for electromagnetic wave to propagate from port i to port j. The (13) shows that wave propagates from port i to port k of component X, then from port k of component X to port l of component Y, then from port l to port j of component Y (See Fig. 7). The



Figure 7. For $i \in X, j \in Y$, there is only one path for wave to propagate from port *i* to port *j*.

time-of-flight of S_{ii} is

$$\operatorname{TOF}(S_{ji}) = \operatorname{TOF}(S_{ki}^{(X)}) + \operatorname{TOF}(S_{jl}^{(Y)})$$
(14)

Self Merging Rule: Let X be an m port with a self loop connected to port l and port k, as shown in Fig. 3.



Figure 8. Self Merging.

After eliminating the self loop, the resultant (m - 2) port has the following S-parameters:

$$S_{ji} = S_{ji}^{(X)} + S_{jl}^{(X)} a_l + S_{jk}^{(X)} a_k \qquad i, j = 1, 2, ..., m - 2$$
(15)

where

$$a_{l} = \frac{1}{\Delta} \left(S_{li}^{(X)} S_{kk}^{(X)} + S_{ki}^{(X)} \left(1 - S_{lk}^{(X)} \right) \right)$$

$$a_{k} = \frac{1}{\Delta} \left(S_{ki}^{(X)} S_{ll}^{(X)} + S_{li}^{(X)} \left(1 - S_{kl}^{(X)} \right) \right)$$

$$\Delta = \left(1 - S_{lk}^{(X)} \right) \left(1 - S_{kl}^{(X)} \right) - S_{ll}^{(X)} S_{kk}^{(X)}$$
(16)

The similar approach could be applied to the self merging process where there are five paths for wave to propagate from port *i* to port *j* (See Fig. 9). The time-of-flight of S_{ii} is



Figure 9. For self merging, there are five paths for wave to propagate from port i to port j.

$$\begin{aligned} \text{TOF}(S_{ji}) &= \min \bigg(\text{TOF}(S_{ji}^{(X)}), \\ &\text{TOF}(S_{ki}^{(X)}) + \text{TOF}(S_{jk}^{(X)}), \\ &\text{TOF}(S_{li}^{(X)}) + \text{TOF}(S_{jl}^{(X)}), \\ &\text{TOF}(S_{ki}^{(X)}) + \text{TOF}(S_{jl}^{(X)}), \\ &\text{TOF}(S_{li}^{(X)}) + \text{TOF}(S_{jk}^{(X)}) \bigg) \end{aligned}$$
(17)

For an arbitrary distributed-lumped network described by the linear components, the Adjoined Merging Rule is used to merge all internal components, and the Self Merging rule is applied to eliminate the self loops introduced by the Adjoined Merging process. As the result, the macromodel, or the voltage transfer function of the network, can be characterized by the S-matrix of the multiport component resulted from the reduction process, together with the S-parameters of the loads. From this reduced network, we can easily get the transfer function. For example, the voltage transfer function of the network shown in Fig. 10 is

$$H(s) = \frac{S_{21}(1+S_o)}{1+S_{11}+S_o(S_{12}S_{21}-S_{11}S_{22}-S_{22})}$$
(18)

where S_o is the s-parameter of the load. The same method could also be applied to (17) for transfer function where there is only one path from the source to the destination.

$$TOF(H(s)) = TOF(S_{21})$$
(19)



Figure 10. A two port network.

From (10), (15) and (17), we can see that the S-parameters in denominators are constants or related to the same ports, so the time-of-flight of the denominators are always zero. Thus, the Corollary 3 can be simplified to:

Corollary 3': If the time-of-flights of the non-zero functions $F_1(s)$ and $F_2(s)$ are $\text{TOF}(F_1(s))$, and $\text{TOF}(F_2(s)) = 0$ respectively, then the time-of-flight of $F_1(s)/F_2(s)$ is

$$TOF(F_1(s)/F_2(s)) = TOF(F_1(s))$$
(20)

Now, we can accurately compute the time-of-flight of a general interconnect system. In order to speed up the reduction process, we expand the S-parameters of components by Taylor series, and derive the formulas for manipulating two Taylor series [10]. Finally, we get the transfer function H(s) in the Taylor series form (that is, the first several moments) with the time-of-flight τ_f . Instead of matching moments of H(s), we match the moments of $H(s) = e^{\tau_f s} H(s)$. The corresponding time domain function h'(t) can be obtained by the Padé approximation [14] or EDPF [5, 10]. Here, we use a mixed exponential function [11]. We approximate h'(t) with the Padé approximation, and match the corresponding moments of unstable poles with the EDPF. Since $H(s) = e^{-\tau_f s} H'(s)$, according to the shifting theorem of Laplace transform, the time domain transfer function is

$$h(t) = h'(t - \tau_f)u(t - \tau_f)$$
(21)

5. Experimental Results

Two testing circuits are presented here to illustrate the efficiency and generality of the macromodel with the timeof-flight captured explicitly. They were executed on a Sun Sparc 1+ workstation.

The first example is a transmission line circuit which was one of benchmarks of 1993 IEEE Multi-Chip Module Conference (MCMC-93) (See Fig. 11). The circuit was



provided by Performance Signal Integrity, Inc. While SPICE3e2 [8] took more than 5 minutes to compute the output waveform $v_o(t)$, our method took 1.2 seconds including 0.7 second for plotting the result. Fig. 12(a) is the analysis result of the 5th order approximation without the capturing of time-of-flight compared to the result of SPICE3e2. The input is a ramp signal with 1.0ns rise time. The experiment result shows that the time-of-flight delay is difficult to be captured with a finite order of exponential. This deficiency affects matching not only on the flat section but also the whole curve. Fig. 12(b) is the result of the 5th order approximation with capturing of the time-of-flight (1.476ns) compared to the result of SPICE3e2. Using the same order approximation, the simulation achieves



Figure 12. The results of the MCMC-93 benchmark generated by SPICE3e2 and our method (a) without the time-of-flight captured (nonTOF) and (b) with the time-of-flight captured (TOF).

much higher accuracy by just capturing the time-of-flight, since it only needs to match the exponentially charging section of the response curves. The extra computation time for capturing the time-of-flight is negligible.

Fig. 13 is a lossy transmission line circuit. While SPICE3e2 [8] took 3.7 seconds to compute the output waveform $v_o(t)$, our method took 1.1 seconds including 0.7 second for plotting the result. Fig. 14(a) is the analysis results of the 4th order approximation without the capturing of time-of-flight capturing compared with SPICE3e2. The input rise time is 0.8ns. Fig. 14(b) is the result of the 4th order approximation with the capturing of time-of-flight (0.582ns) compared with SPICE3e2. Again, the accuracy of response curves, not only on the flat section, but also on the whole curve has been greatly improved.



Figure 13. A transmission line circuit.



Figure 14. The results of the lossy line circuit generated by SPICE3e2 and our method (a) without the time-of-flight captured (non-TOF) and (b) with the time-of-flight captured (TOF).

6. Conclusions

In this paper, we point out the deficiency of previous definition of the time-of-flight and give a precise definition of the time-of-flight together with some properties. Based on the new definition and the properties of the time-offlight, we can easily capture the time-of-flight delay for general interconnect networks during the reduction while using lower order model to evaluate the charging delay. The experimental results show that the accuracy of the transient analysis, because of capturing the time-of-flight delay, is greatly improved without significantly increasing computing time.

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References

[1] N. Balabanian, T. A. Bickart, <u>Electrical Network Theory</u>, ch. 6, Malabar, Florida: Robert E. Krieger, 1985.

- [2] J. E. Bracken, V. Raghavan, and R. A. Rohrer, "Simulating Distributed Elements with Asymptotic Waveform Evaluation," *IEEE MTT-S International Microwave Symposium Digest*, pp. 1337-1340, 1992.
- [3] J. E. Bracken, V. Raghavan, and R. A. Rohrer, "Extension of the Asymptotic Waveform Evaluation Technique with the Method of Characteristics," Technical Digest of the *IEEE International Conference on Computer-Aided Design*, pp. 71-75, Nov. 1992.
- [4] F. Y. Chang, "Waveform Relaxation Analysis of Nonuniform Lossy Transmission Lines Characterized with Frequency-Dependent Parameters," *IEEE Trans. on Circuits* and Systems, vol. CAS-38, pp. 1484-1500, Dec. 1991.
- [5] F. Y. Chang, "Transient Simulation of Nonuniform Coupled Lossy Transmission Lines Characterized with Frequency-Dependent Parameters, Part II: Discrete-Time Analysis," *IEEE Trans. on Circuits and Systems*, vol. CAS-39, pp. 907-927, Nov. 1992.
- [6] J. Dobrowolski, <u>Introduction to Computer Methods for</u> <u>Microwave Circuit Analysis and Design</u>, Artech House, 1991.
- [7] W. C. Elmore, "The Transient Response of Damped Linear Networks with Particular Regard to Wideband Amplifier," J. Applied Physics, 19(1), 1948.
- [8] B. Johnson, T. Quarles, A. R. Newton, D. O. Pederson and A. Sangiovanni-Vincentelli, "SPICE3 Version 3e User's Manual," University of California, Berkeley, April 1991.
- [9] S. Y. Kim, N. Gopal and L. T. Pillage, "Finite-Pole Macromodels of Transmission Lines for Circuit Simulation," *Proceedings of the 1993 Custom Integrated Circuits Conference*.
- [10] H. Liao, W. Dai, R. Wang, and F. Y. Chang, "S-Parameter Based Macro Model of Distributed-Lumped Networks Using Exponentially Decayed Polynomial Function," *Proceedings of the 30th ACM/IEEE Design Automation Conference*, pp. 726-731, June 1993.
- [11] H. Liao and W. Dai, "Transient Analysis of Interconnects with Nonlinear Driver Using Mixed exponential Function Approximation," *Technique report of University of California at Santa Cruz*, UCSC-CRL-93-44, Oct. 1993.
- [12] H. Liao and W. Dai, "Capturing Time-of-Flight Delay for Transient Analysis Based on Scattering Parameter Macromodel," *Technique report of University of California at Santa Cruz*, UCSC-CRL-94-11, April. 1994.
- [13] S. Lin, and E. S. Kuh, "Transient Simulation of Lossy Interconnects Based on the Recursive Convolution Formulation," *IEEE Trans. on Circuits and Systems*, vol. CAS-39, pp. 879-892, Nov. 1992.
- [14] L. T. Pillage, and R. A. Rohrer, "Asymptotic Waveform Evaluation for Timing Analysis," *IEEE Trans. on CAD*, vol. CAD-9, April 1990.
- [15] J. White, L. Pillage, and J. Cohn, "Computer-Aided Analysis and Design of Interconnect" *Tutorial of IEEE/ACM International Conference on Computer-Aided Design*, Nov. 1993.