An Efficient and Globally Convergent Homotopy Method for Finding DC Operating Points of Nonlinear Circuits

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Abstract— Finding DC operating points of nonlinear circuits is an important problem in circuit simulation. The Newton-Raphson method employed in SPICE-like simulators often fails to converge to a solution. To overcome this convergence problem, homotopy methods have been studied from various viewpoints. There are several types of homotopy methods, one of which succeeded in solving bipolar analog circuits with more than 20000 elements with the theoretical guarantee of global convergence. In this paper, an improved version of the homotopy method is proposed that can find DC operating points of practical nonlinear circuits smoothly and efficiently. It is also shown that the proposed method can be easily implemented on SPICE without programming.

I. Introduction

Finding DC operating points of nonlinear circuits is an important problem in circuit simulation. SPICE-like circuit simulators, which are widely used in LSI design, employ the Newton-Raphson (NR) method for solving modified nodal (MN) equations. However, the NR method or its variants often fails to converge to a solution unless the initial point is sufficiently close to the solution. Therefore, many circuit designers experience difficulties in finding DC operating points, especially for bipolar analog integrated circuits.

To overcome this convergence problem, globally convergent homotopy methods have been studied by many researchers from various viewpoints. By these studies, the application of the homotopy methods in practical circuit simulation has been remarkably developed, and the homotopy method termed the Newton homotopy (NH) method succeeded in solving bipolar analog circuits with more than 20000 elements (that belong to a class of the largest-scale circuits available with the current bipolar analog LSI technology) with the theoretical guarantee of global convergence [1]–[5]. However, since the NH method is globally convergent only when we choose an initial point on which the uniform passivity holds, we cannot choose a *good* initial point (e.g., point in the forward active operation region of transistors).

The Newton-fixed-point homotopy (NFPH) method is an improved version of the NH method [6],[7]. In this method, we can trace a solution curve from a good initial point, which often makes the solution curve short and the algorithm efficient. However, the auxiliary equation of this method contains a linear function that has no relation to the original nonlinear function, which sometimes causes complicated movement of solution curves, especially in the neighbourhood of $\lambda = 1$.

As another efficient approach of the homotopy method, the variable gain homotopy (VGH) method is well-known [8], which is an extension of the homotopy method termed the fixed-point homotopy (FPH) method [9]. Since this method includes the excellent idea of variable gain, solution curves often become smooth. However, in this method, we sometimes have to trace a solution curve from an initial point far from the solution; namely, the initial state is sometimes far from the normal operation of tansistor circuits. In order to solve large-scale circuits more efficiently, it is necessary to develop a more efficient homotopy method.

In this paper, an efficient homotopy method termed the *variable gain Newton homotopy (VGNH) method* is proposed that is based on the idea of the NFPH method and that of the VGH method. The proposed method has the following advantages: i) The auxiliary equation is closely related to the original nonlinear equation. ii) Since this method is globally convergent for any initial point, we can choose a good initial point. iii) The idea of variable gain is introduced. Therefore, we can trace solution curves smoothly and efficiently. By numerical examples, it is shown that the proposed method finds DC operating points of practical transistor circuits more efficiently than the conventional methods. It is also shown that the proposed method can be easily implemented on SPICE without programming.

II. Homotopy Methods for MN Equations

We first review the homotopy methods for solving systems of nonlinear equations of the form:

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}, \quad \boldsymbol{f} : \boldsymbol{R}^n \to \boldsymbol{R}^n. \tag{1}$$

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In the MN equation, (1) is written as follows [2],[4]:

$$\begin{aligned} \boldsymbol{f}_g(\boldsymbol{v},\boldsymbol{i}) &\triangleq \boldsymbol{D}_g\boldsymbol{g}(\boldsymbol{D}_g{}^T\boldsymbol{v})+\boldsymbol{D}_E\boldsymbol{i}+\boldsymbol{J}=\boldsymbol{0} \\ \boldsymbol{f}_E(\boldsymbol{v},\boldsymbol{i}) &\triangleq \boldsymbol{D}_E{}^T\boldsymbol{v}-\boldsymbol{E}=\boldsymbol{0}, \end{aligned} \tag{2}$$

where $\boldsymbol{v}\in\boldsymbol{R}^N$ is the variable vector denoting the node voltages to the datum node, $\boldsymbol{i}\in\boldsymbol{R}^M$ is the variable vector denoting the branch currents of the independent voltage sources, $\boldsymbol{g}:\boldsymbol{R}^K\rightarrow\boldsymbol{R}^K$ is a VCCS type continuous function, $\boldsymbol{D}_g$ and $\boldsymbol{D}_E$ are $N\times K$ and $N\times M$ (resp.) reduced incidence matrices, $\boldsymbol{J}\in\boldsymbol{R}^N$ and $\boldsymbol{E}\in\boldsymbol{R}^M$ are source vectors, $\boldsymbol{f}=(\boldsymbol{f}_g,\boldsymbol{f}_E)^T:\boldsymbol{R}^n\rightarrow\boldsymbol{R}^n$, $\boldsymbol{x}=(\boldsymbol{v},\boldsymbol{i})^T\in\boldsymbol{R}^n$, and $n=N+M$.

In transistor circuits, the branch $\boldsymbol{g}$ is composed of transistors, diodes, resistors, etc. The relationship between the branch voltage vector $\boldsymbol{v}_q=(v_{\mathrm{be}},v_{\mathrm{bc}})^T$ and the branch current vector $\boldsymbol{i}_q=(i_{\mathrm{e}},i_{\mathrm{c}})^T$ of a bipolar junction transistor (BJT), for instance, is described by the Ebers-Moll model as follows:

$$\boldsymbol{i}_q(\boldsymbol{v}_q)=\boldsymbol{T}\boldsymbol{q}(\boldsymbol{v}_q), \tag{3}$$

where

$$\boldsymbol{T}=\begin{bmatrix} 1 & -\alpha_r \\ -\alpha_f & 1 \end{bmatrix} \tag{4}$$

and

$$\boldsymbol{q}(\boldsymbol{v}_q)=\begin{bmatrix} m_e(\exp(n_e v_{\mathrm{be}})-1) \\ m_c(\exp(n_c v_{\mathrm{bc}})-1) \end{bmatrix}. \tag{5}$$

Also, i_{e} (i_{c}) denotes the emitter (collector) current and v_{be} (v_{bc}) denotes the base to emitter (collector) voltage, respectively. The model parameters $\alpha_f,\alpha_r,m_e,m_c,n_e$, and n_c are required to satisfy the passivity, no-gain, and reciprocity conditions [10].

In the homotopy methods [9], we consider an auxiliary equation $\boldsymbol{f}^0(\boldsymbol{x})=\boldsymbol{0}$ with a known solution $\boldsymbol{x}^0$ (or a solution easily obtained) and define a homotopy function:

$$\boldsymbol{h}(\boldsymbol{x},\lambda)=\lambda\boldsymbol{f}(\boldsymbol{x})+(1-\lambda)\boldsymbol{f}^0(\boldsymbol{x}), \tag{6}$$

where $\lambda\in[0,1]$ is the homotopy parameter. Then, the solution curve (often called the path) of the homotopy equation:

$$\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{0} \tag{7}$$

is traced from the initial point $(\boldsymbol{x}^0,0)$ at $\lambda=0$. If the solution curve reaches the $\lambda=1$ hyperplane at $(\boldsymbol{x}^*,1)$, then a solution $\boldsymbol{x}^*$ of (1) is obtained.

There are several types of homotopy methods for solving MN equations. The NFPH method [6],[7] uses the homotopy function:

$$\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{f}(\boldsymbol{x})-(1-\lambda)\boldsymbol{f}(\boldsymbol{x}^0)+(1-\lambda)\boldsymbol{A}(\boldsymbol{x}-\boldsymbol{x}^0), \tag{8}$$

where $\boldsymbol{A}$ is an $n\times n$ matrix represented as follows:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{D}_g\boldsymbol{G}_{FP}\boldsymbol{D}_g^T & \boldsymbol{0} \\ \boldsymbol{0} & -R_{FP}\boldsymbol{1}_M \end{bmatrix}. \tag{9}$$

In (9), $\boldsymbol{G}_{FP}$ is a positive semi-definite diagonal matrix whose diagonal elements are positive and others are zero. Also, R_{FP} is a scalar positive value and $\boldsymbol{1}_M$ denotes an $M\times M$ identity matrix. Note that the auxiliary function $\boldsymbol{f}^0(\boldsymbol{x})$ at $\lambda=0$ contains a linear function that has no relation to the original nonlinear function $\boldsymbol{f}(\boldsymbol{x})$.

The VGH method [8] uses the homotopy function:

$$\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{f}(\boldsymbol{x},\lambda\boldsymbol{\alpha})+(1-\lambda)\boldsymbol{G}(\boldsymbol{x}-\boldsymbol{a}), \tag{10}$$

where $\boldsymbol{\alpha}$ is a vector consisting of forward current gains α_f and reverse ones α_r of transistors, $\boldsymbol{a}$ is a random vector, and $\boldsymbol{G}$ is an $N\times N$ diagonal matrix. The VGH method is a two-stage procedure. In Phase 1, the initial point $\boldsymbol{x}^0$ that satisfies $\boldsymbol{h}(\boldsymbol{x},0)=\boldsymbol{0}$ is computed by the modified NR method. In Phase 2, the solution curve of $\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{0}$ is traced from $(\boldsymbol{x}^0,0)$. In Phase 1, the circuit described by $\boldsymbol{h}(\boldsymbol{x},0)=\boldsymbol{0}$ contains diodes as only nonlinear elements, hence it has a unique solution.

Under some regularity assumptions of $\boldsymbol{h}$, the solution curve of $\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{0}$ is guaranteed to reach the $\lambda=1$ hyperplane if the uniqueness condition at $\lambda=0$ and the boundary free condition hold [4],[9]. Several homotopy methods including the NFPH method are proven to be globally convergent for MN equations [1],[4],[6],[7]; namely, the solution curve of $\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{0}$ is guaranteed to reach the $\lambda=1$ hyperplane.

III. Proposed Method

In this section, we propose a new homotopy method that is not only globally convergent but also very efficient. We first consider the following homotopy function:

$$\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{f}(\boldsymbol{x},\lambda\boldsymbol{\alpha})-(1-\lambda)\boldsymbol{f}(\boldsymbol{x}^0,0\cdot\boldsymbol{\alpha}), \tag{11}$$

where $0\cdot\boldsymbol{\alpha}$ implies the product of 0 and $\boldsymbol{\alpha}$. Note that this homotopy function includes the concept of variable gain. If we consider a circuit described by $\boldsymbol{h}(\boldsymbol{x},\lambda)=\boldsymbol{0}$, then each transistor of the circuit can be described by (3) with $\boldsymbol{T}$ replaced by

$$\boldsymbol{T}_\lambda=\begin{bmatrix} 1 & -\lambda\alpha_r \\ -\lambda\alpha_f & 1 \end{bmatrix}. \tag{12}$$

If we put

$$\tilde{\boldsymbol{T}}=\begin{bmatrix} 0 & \alpha_r \\ \alpha_f & 0 \end{bmatrix}, \tag{13}$$

then

$$\boldsymbol{T}_\lambda=\boldsymbol{T}+(1-\lambda)\tilde{\boldsymbol{T}} \tag{14}$$

holds.

Next, consider the following function:

$$\tilde{\boldsymbol{f}}(\boldsymbol{x})\triangleq\begin{bmatrix} \boldsymbol{D}_g\tilde{\boldsymbol{g}}(\boldsymbol{D}_g^T\boldsymbol{v}) \\ \boldsymbol{0} \end{bmatrix}, \tag{15}$$

where the components $\tilde{g}_i$ ($i=1,2,\cdots,K$) of $\tilde{\boldsymbol{g}}=(\tilde{g}_1, \tilde{g}_2,\cdots,\tilde{g}_K)^T$ are defined as follows:

1. If g_i and g_{i+1} are a pair of transistor branches, that is,
$$\begin{bmatrix} g_i \\ g_{i+1} \end{bmatrix} = \boldsymbol{T}\boldsymbol{q}(\boldsymbol{v}_q), \tag{16}$$
then the corresponding function $\tilde{\boldsymbol{g}}_q = (\tilde{g}_i, \tilde{g}_{i+1})^T$ is
$$\begin{bmatrix} \tilde{g}_i \\ \tilde{g}_{i+1} \end{bmatrix} = \tilde{\boldsymbol{T}}\boldsymbol{q}(\boldsymbol{v}_q). \tag{17}$$

2. If g_i is not a transistor branch, then
$$\tilde{g}_i = 0. \tag{18}$$

Then, from (14)–(18), it is easily seen that $\boldsymbol{f}(\boldsymbol{x}, \lambda\boldsymbol{\alpha}) = \boldsymbol{f}(\boldsymbol{x}) + (1-\lambda)\tilde{\boldsymbol{f}}(\boldsymbol{x})$ and $\boldsymbol{f}(\boldsymbol{x}^0, 0\cdot\boldsymbol{\alpha}) = \boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$ hold. Hence, (11) can be rewritten as:
$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{f}(\boldsymbol{x}) + (1-\lambda)\tilde{\boldsymbol{f}}(\boldsymbol{x}) - (1-\lambda)(\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)) \tag{19}$$
and we have
$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{f}(\boldsymbol{x}) - (1-\lambda)\boldsymbol{f}(\boldsymbol{x}^0) + (1-\lambda)(\tilde{\boldsymbol{f}}(\boldsymbol{x}) - \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)). \tag{20}$$
Note that (20) is equivalent to (11). In this paper, we propose a homotopy method using the homotopy function (11) or (20). From the form of (11), the proposed method may be called the variable gain Newton homotopy (VGNH) method.

The basic VGH method requires some modifications to the model subroutines such as (12), but as seen from (20), the proposed method requires only additional subroutines of $\tilde{\boldsymbol{T}}$. Thus, the proposed method can be implemented on the SPICE-like simulators with no modification to the existing model subroutines.

For the global convergence property of the proposed method, the following theorem holds.

Theorem 1 Consider the homotopy function defined by (11) or (20). Assume that $\boldsymbol{g}$ is uniformly passive [4] on certain points. Then, for any initial point $\boldsymbol{x}^0 \in \boldsymbol{R}^n$, the solution curve of $\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{0}$ starting from $(\boldsymbol{x}^0, 0)$ reaches $\lambda = 1$. □

Proof : To prove the theorem, it is sufficient to show that i) $\boldsymbol{x}^0$ is the unique solution of $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$, and ii) $\boldsymbol{h}$ is boundary free [4],[9]. i) Consider the circuit described by $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$. Since $\boldsymbol{T}_\lambda = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ holds at $\lambda = 0$, this circuit has diodes as only nonlinear elements as well as in Phase 1 of the VGH method. Hence, this circuit has a unique solution, which implies that the solution of $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$ is unique. ii) It is evident that the branch $\boldsymbol{g} + (1-\lambda)\tilde{\boldsymbol{g}}$ satisfies the uniform passivity on certain points for all $\lambda \in [0, 1]$. Hence, following the proofs discussed in [1] and [4], it is trivial to show that $\boldsymbol{h}$ is boundary free. Thus, the global convergence of the proposed method is guaranteed for any initial point $\boldsymbol{x}^0 \in \boldsymbol{R}^n$. □

Note that a fairly general class of resistive elements including BJTs, diodes, tunnel diodes, and positive linear resistors are known to be uniformly passive on certain points [4]. Thus, the uniform passivity is a very mild condition.

Next, we discuss the computational efficiency of the proposed method, considering the factors that degrade the efficiency in the conventional methods. In the VGH method, the initial point $\boldsymbol{x}^0$ in Phase 2 is obtained by solving a circuit that contains diodes as only nonlinear elements. However, since the structure of such a circuit often makes the operation of some transistors not be the normal (foward active) operation of transistor circuits, the initial point obtained in Phase 1 is sometimes far from the solution. In the NFPH method, a good initial point can be used as discussed in [7]. However, as stated before, the auxiliary function contains a linear function that has no relation to the original nonlinear function. Moreover, this method requires relatively large values of some elements of $\boldsymbol{G}_{FP}$ in (9) to guarantee the uniqueness condition at $\lambda = 0$ [7]. Such linear function with large $\boldsymbol{G}_{FP}$ sometimes causes complicated movement of solution curves, especially in the neighbourhood of $\lambda = 1$ [6],[7].

We show here that the proposed method is free from the difficulties of the VGH method and the NFPH method. First, since $\boldsymbol{h}(\boldsymbol{x}^0, 0) = \boldsymbol{0}$ holds for any $\boldsymbol{x}^0$, we can choose a good initial point as discussed in [7]. Secondly, the homotopy function (11) or (20) contains no linear function, and the auxiliary equation $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$ is closely related to the original nonlinear equation $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}$. Hence, the proposed method is free from the problem of the NFPH method mentioned above. Moreover, since the proposed method includes the concept of variable gain, it is expected that the solution curves become smooth and short.

We now propose an efficient variation of the proposed method. As has been discussed, $\boldsymbol{x}^0$ is the unique solution of $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$ defined by (20), independent of the circuit parameters or topologies. However, it is well-known among circuit designers that many practical transistor circuits cannot have multiple solutions under the condition $\alpha_f \leq 0.5$ $(\beta_f = \alpha_f/(1-\alpha_f) \leq 1)$ [10]. Considering this property, we can propose a more practical homotopy function:
$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{f}(\boldsymbol{x}) - (1-\lambda)\boldsymbol{f}(\boldsymbol{x}^0) + (1-\lambda)\frac{(\tilde{\boldsymbol{f}}(\boldsymbol{x}) - \tilde{\boldsymbol{f}}(\boldsymbol{x}^0))}{2}, \tag{21}$$
which is obtained by replacing $\boldsymbol{T}_\lambda$ in (14) with
$$\begin{aligned} \boldsymbol{T}_\lambda &= \boldsymbol{T} + (1-\lambda)\frac{\tilde{\boldsymbol{T}}}{2} \\ &= \begin{bmatrix} 1 & -\alpha_r \\ -\alpha_f & 1 \end{bmatrix} + (1-\lambda)\begin{bmatrix} 0 & \frac{\alpha_r}{2} \\ \frac{\alpha_f}{2} & 0 \end{bmatrix}. \end{aligned} \tag{22}$$
This is the second homotopy function proposed in this paper, where the current gains change from $\boldsymbol{\alpha}/2$ to $\boldsymbol{\alpha}$,

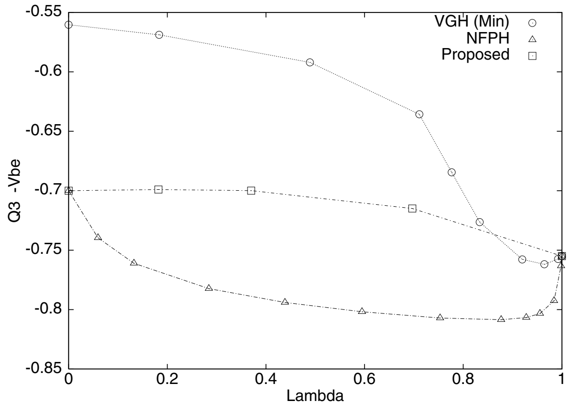

Fig. 1. Solution curves for HVRef.

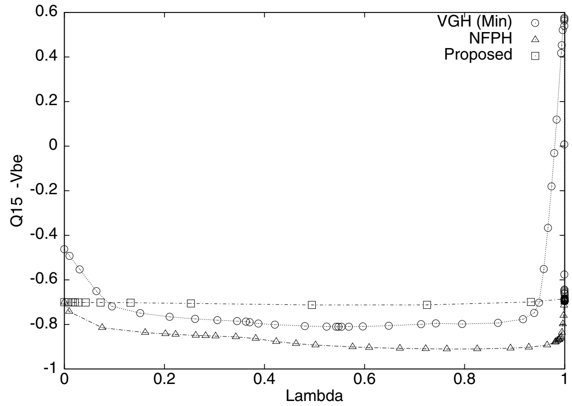

Fig. 2. Solution curves for μA741.

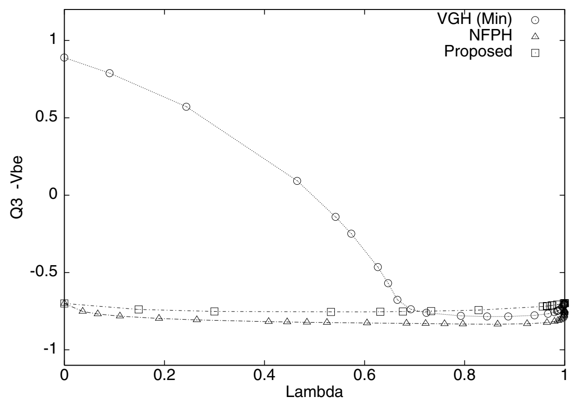

Fig. 3. Solution curves for RegCkt.

not from $\mathbf{0}$ to $\boldsymbol{\alpha}$. Hence, it is expected that $\boldsymbol{h}(\boldsymbol{x}, 0) = \mathbf{0}$ defined by (21) is closer to the original nonlinear equation than that defined by (20). Thus, from the practical viewpoint, we recommend the following algorithm.

1. We first apply the homotopy method using the homotopy function (21).
2. If the above method happens to fail, then we apply the homotopy method using the homotopy function (20), whose global convergence is theoretically guaranteed.

IV. Numerical Examples

We implemented the proposed method on a Sun Blade 2000 (UltraSPARC-III Cu 1.2GHz) and have confirmed the effectiveness of the proposed method using many practical transistor circuits. In all of the numerical experiments, the proposed method was the most efficient. In this section, we show the results applied to five types of practical transistor circuits widely used in analog LSIs; namely, the hybrid voltage reference circuit (HVRef) [6]–[8], a basic two-stage operational amplifier (2sOA) [7], a six-stage limiting amplifier (6sLA) [7], a high-gain operational amplifier μA741 that consists of 29 elements including 22 BJTs, and a regulator circuit (RegCkt) with an output voltage of 4.2 V that is used in bipolar LSIs and consists of 41 elements including 24 BJTs.

In the numerical experiments, we used the typical set of model parameters as those used in [7] for BJT models. We chose the initial points in the forward active operation region for all transistors [7]. It is natural to use $\boldsymbol{v}_q^0 = (-0.7, 0)^T$ as a typical forward active state for silicon npn transistors and $\boldsymbol{v}_q^0 = (0.7, 0)^T$ for pnp. We also used the spherical method [1],[2] for tracing solution curves.

Figs. 1–3 show the solution curves for HVRef, μA741, and RegCkt. In these figures, the emitter to base voltage $-v_{\mathrm{be}}$ of a certain BJT is plotted, where marks indicate the steps. In each step, a system of $n+1$ nonlinear equations is solved by the NR method. From these figures, it is seen that the proposed method traces solution curves more smoothly and efficiently than the conventional methods (VGH and NFPH).

For the comparison of computational efficiency, we summarize in Table I the number of steps and the total number of Newton iterations of the above three methods. For the VGH method, the minimum (Min) and maximum (Max) numbers are shown in the hundred trials, and the left-hand numbers in the parentheses indicate the modified NR iterations in Phase 1. From the viewpoint of Newton iterations, the proposed method is several times more efficient than the VGH (Min) method and the NFPH method.

V. Extension to Circuits Containing Tunnel Diodes

In this section, we extend the proposed method to circuits containing tunnel diodes.

In general, the v-i characteristic of a tunnel diode, which will be written by $i = g(v)$, is represented by a polynomial:

$$g(v) = av^3 - bv^2 + cv, \qquad a, b, c > 0. \tag{23}$$

TABLE I
COMPARISON OF COMPUTATIONAL EFFICIENCY.

Method		VGH (Min)		VGH (Max)		NFPH		Proposed	
Circiut	n	Number of steps (Number of Newton iterations)							
HVRef	41	9	(11+25)	35	(13+99)	11	(44)	4	(16)
2sOA	42	4	(11+17)	21	(9+71)	3	(13)	3	(8)
6sLA	80	14	(10+46)	48	(10+152)	3	(12)	3	(9)
μA741	95	72	(16+177)	213	(17+494)	47	(173)	12	(33)
RegCkt	95	36	(16+99)	60	(16+161)	46	(128)	22	(67)

TABLE II
COMPARISON OF CONVERGENCE RATE (%).

n	NH	Proposed
10	29	100
20	15	100
50	1	100
100	0	100

TABLE III
COMPARISON OF COMPUTATINAL EFFICIENCY.

	NH method			Proposed method		
n	S	L	T (s)	S	L	T (s)
500	5 040	1 570	87	22	83	0.5
1000	16 040	4 480	1 591	52	153	8
1500	57 838	8 026	15 037	88	215	51
2000	83 941	12 352	55 410	120	282	190
2500	–	–	–	151	339	416
3000	–	–	–	145	389	658
3500	–	–	–	166	443	1 059
4000	–	–	–	176	495	2 977
4500	–	–	–	251	544	3 278
5000	–	–	–	236	600	3 934

This function is not monotone because of the existence of the second term $-bv^2$. Hence, we consider the following function:

$$g(v,\lambda) = av^3 - \lambda bv^2 + cv, \qquad a, b, c > 0. \tag{24}$$

At $\lambda = 0$, $g(v, 0)$ is a monotone function and satisfies the uniform passivity on any point. Hence, the uniqueness condition at $\lambda = 0$ holds for any initial point. Moreover, the function defined by (24) changes continuously from a monotone function $g(v, 0)$ to the original function $g(v)$ as λ changes from 0 to 1, which often makes the solution curve smooth and short. This idea may be considered as an extension of the VGH method to tunnel diode circuits. Note that (24) can be realized by using $\tilde{g}(v) = -bv$ in (15) and (20).

Now we show some numerical examples. We consider the circuit containing n tunnel diodes discussed in [5].

We first compare the global convergence property of the NH method and the proposed method. Table II compares the convergence rate when we applied the two methods from randomly chosen one hundred initial points for n = 10, 20, 50, and 100. As seen from the table, the NH method often fails to converge; more precisely, the solution curve of the NH method often returned back to $\lambda = 0$. This is because the global convergence of the NH method is guaranteed only when we choose an initial point on which the uniform passivity hold [1],[4]. However, the proposed method always converged to a solution from any initial point as guaranteed in Theorem 1.

We next compare the computational efficiency of the NH method[1] and the proposed method in Table III, where S denotes the total number of steps, L denotes the arc-length of solution curves, T (s) denotes the computation time, and "–" denotes that it could not be computed in one day. We used the initial point $\boldsymbol{x}^0 = \mathbf{0}$, on which $g(v)$ is uniformly passive. Both methods found the same solution for all n. It is seen that the proposed method is much more efficient than the NH method, and could solve this circuit for n = 5000 in about one hour. It is also seen that the arc-length of solution curves is much smaller in the proposed method. Typical examples are shown in Figs. 4 and 5, where Fig. 4 shows the solution curve of the NH method for n = 200 and Fig. 5 shows the solution curve of the proposed method for n = 5000. In both figures, the vertical line denotes the arc-length of the solution curve. In Fig. 4, we can see a complicated solution curve with many sharp turning points. Such solution curves have often been observed when we applied the conventional homotopy methods to large-scale circuits [1]. However, In Fig. 5, we can see a smooth and short solution curves although n is large.

[1]In the comparison, we chose the NH method because the VGH method cannot be applied to this circuit, and the NFPH method seems to be less efficient than the NH method for this circuit when we used $\boldsymbol{x}^0 = \mathbf{0}$.

VI. IMPLEMENTATION OF THE VGNH METHOD ON SPICE WITHOUT PROGRAMMING

Thus, the proposed method is not only globally convergent for any initial point but also efficient because we can use good initial points and the solution curves tend to become smooth and short. In a sense, the proposed method has all the advantages of the NH, NFPH, and VGH methods, and is free from the difficulties of these

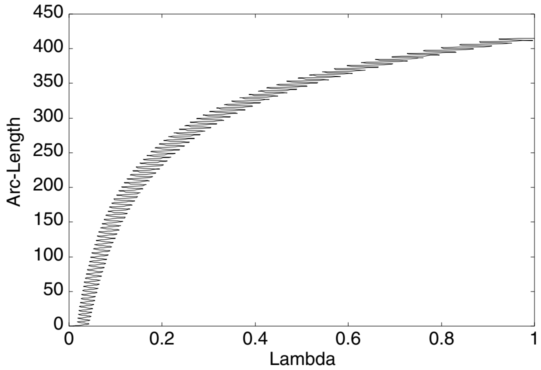

Fig. 4. Solution curve of the NH method ($n = 200$).

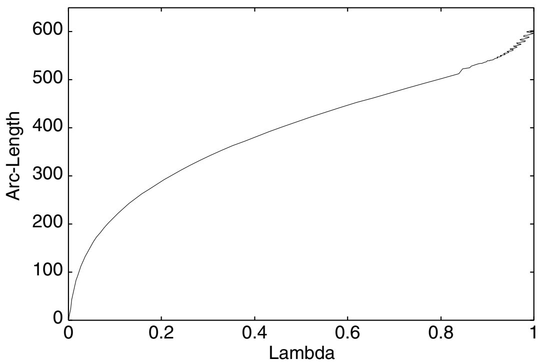

Fig. 5. Solution curve of the proposed method ($n = 5000$).

methods.

However, the programming of sophisticated homotopy methods is often difficult for non-experts or beginners. In this section, we propose an effective method for implementing the VGNH method on SPICE using the idea of *SPICE-oriented numerical methods* [1],[11]–[13].

In the VGNH method, we trace the solution curve of $\boldsymbol{h}(\boldsymbol{x}, \lambda) = \mathbf{0}$ where $\boldsymbol{h}$ is defined by (19). The solution curve can be traced by integrating a system of the algebraic-differential equations:

$$\boldsymbol{f}(\boldsymbol{x}) + (1-\lambda)\tilde{\boldsymbol{f}}(\boldsymbol{x}) - (1-\lambda)(\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)) = \mathbf{0} \quad \text{(25a)}$$

$$\sum_{i=1}^{m} \left(\frac{dv_{\mathrm{be}i}}{ds}\right)^2 + \left(\frac{d\lambda}{ds}\right)^2 = 1 \quad \text{(25b)}$$

starting from $(\boldsymbol{x}^0, 0)$ [1],[11]–[13], where m denotes the number of transistors contained in the circuit, s denotes the arc-length of the solution curve starting from $(\boldsymbol{x}^0, 0)$, and $v_{\mathrm{be}i}$ denotes the base-emitter voltage of the ith transistor. Note that the points on the solution curve are considered as functions of s, namely, they can be written as $(\boldsymbol{x}(s), \lambda(s))$.

In the approach of the SPICE-oriented numerical methods, we consider a circuit described by (25). Then, we perform the transient analysis of SPICE to the circuit

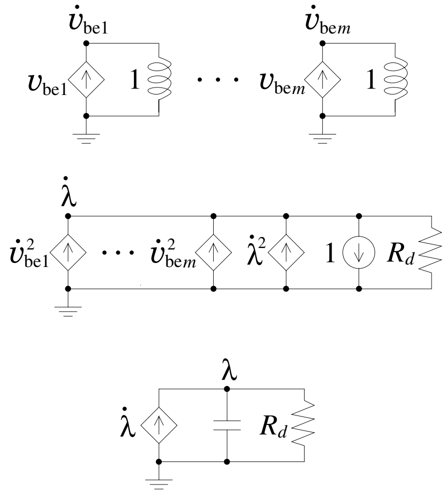

Fig. 6. Circuits that describe (25b).

starting from $(\boldsymbol{x}^0, 0)$, by which numerical integration is applied to (25) and the solution curve of (25a) is traced.

As discussed in [1] and [11]–[13], (25b) is described by the circuits shown in Fig. 6. In this figure, $\dot{v}_{\mathrm{be}i}$ or $\dot{\lambda}$ denotes a node voltage that is independent of $v_{\mathrm{be}i}$ or λ but is equal to $dv_{\mathrm{be}i}/ds$ or $d\lambda/ds$ as a result, respectively. Such circuits are called *path following circuits*.

Next, we consider a circuit that is described by (25a). However, this is not an easy task because of the following reasons.

1. In the VGNH method, we first determine a good initial point $\boldsymbol{x}^0$. It is natural to choose $\boldsymbol{x}^0$ so that $\boldsymbol{v}_q = (v_{\mathrm{be}}, v_{\mathrm{bc}})^T$ becomes a point in the forward active operation region for all transistors. However, since $\boldsymbol{v}_q$ is a vector consisting of branch voltages but $\boldsymbol{x}$ is a vector consisting of node voltages and branch currents of the independent voltage sources, we have to calculate the initial point $\boldsymbol{x}^0$ such that $\boldsymbol{v}_q$ becomes a point in the forward active operation region.

2. We have to determinte the constant term $\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$ in (25a), which cannot be obtained by substituting $\boldsymbol{x}^0$ to $\boldsymbol{f}(\boldsymbol{x})$ or $\tilde{\boldsymbol{f}}(\boldsymbol{x})$ because the formulas of $\boldsymbol{f}(\boldsymbol{x})$ do not appear explicitly in SPICE.

Therefore, the proposed implementation method consists of two phases.

A. Determination of the initial point $\boldsymbol{x}^0$ and the constant term $\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$.

In Phase 1 of the proposed method, we first set $\boldsymbol{v}_q$ of all transistors in the forward active operation region [e.g., $\boldsymbol{v}_q = (0.7, 0)^T$]. Let such a point be $\boldsymbol{v}_q^0 = (V_{\mathrm{be}}^0, V_{\mathrm{bc}}^0)^T$. Then, we connect the independent voltage sources V_{be}^0 and V_{bc}^0 to each transistor as shown in Fig. 7.

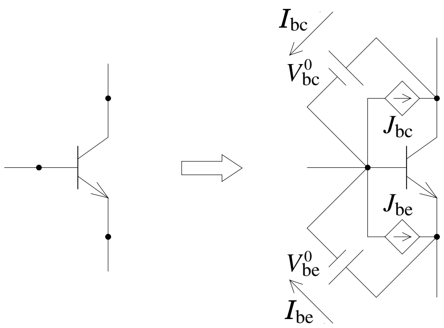

Fig. 7. The initial circuit for determining the initial point $\boldsymbol{x}^0$ and the constant term $\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$.

We next connect the controlled current sources $J_{\rm be}$ and $J_{\rm bc}$ to each transistor as shown in Fig. 7, where currents of $J_{\rm be}$ and $J_{\rm bc}$ are described by $\tilde{g}_i$ and $\tilde{g}_{i+1}$ in (17), respectively. By the definition of $\tilde{\boldsymbol{f}}(\boldsymbol{x})$, it is easily seen that connecting these controlled sources is equivalent to adding $\tilde{\boldsymbol{f}}(\boldsymbol{x})$ to the left-hand side of the original MN equations $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}$. Such a circuit where two independent voltage sources and two controlled current sources are connected to each transistor of the original circuit is called the *initial circuit*.

Then, we solve the initial circuit by the DC analysis of SPICE. Since the initial circuit is essentially a linear circuit, it can be solved by the DC analysis of SPICE. Let the solution of the initial circuit be $\boldsymbol{x}^0$. Since $\boldsymbol{v}_q = (V_{\rm be}^0, V_{\rm bc}^0)^T$ holds in $\boldsymbol{x}^0$, it can be used as a good initial point of the VGNH method. Moreover, since the original circuit is described by $\boldsymbol{f}(\boldsymbol{x})$ and the controlled sources $J_{\rm be}$ and $J_{\rm bc}$ are described by $\tilde{\boldsymbol{f}}(\boldsymbol{x})$, considering the Kirchhoff's current law, it is easily seen that the currents of the independent voltage sources $V_{\rm be}^0$ and $V_{\rm bc}^0$ (that are denoted by $I_{\rm be}$ and $I_{\rm bc}$ in Fig. 7) give the linear term $\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$. Note that it is sufficient to consider the constant term only at the nodes where transistors are connected, because at the node n_j where transistors are not connected, $f_j(\boldsymbol{x}^0) = 0$ and $\tilde{f}_j(\boldsymbol{x}^0) = 0$ hold.

Thus, by solving the initial circuit, the initial point $\boldsymbol{x}^0$ and the constant term $\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$ are obtained.

B. Solving circuits that describe (25).

Now it is clear that (25a) is described by a circuit as shown in Fig. 8, where four controlled current sources are connected to each transistor. Namely, by connecting $(1-\lambda)J_{\rm be}$ and $(1-\lambda)J_{\rm bc}$, $(1-\lambda)\tilde{\boldsymbol{f}}(\boldsymbol{x})$ is described, and by connecting $(1-\lambda)I_{\rm be}$ and $(1-\lambda)I_{\rm bc}$, $-(1-\lambda)(\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0))$ is described. In Phase 2 of the proposed method, we perform the transient analysis of SPICE to this circuit together with the path following circuits shown in Fig. 6, and trace the solution curve of (25a).

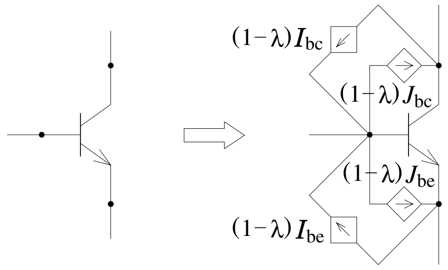

Fig. 8. The circuit that describes (25a).

TABLE IV
COMPARISON OF COMPUTATION TIME.

		Program		SPICE	
Circuit	n	S	T (s)	S	T (s)
HVRef	41	21	0.117	21	0.060
2sOA	42	19	0.133	19	0.020
6sLA	80	19	0.500	19	0.120
μA741	95	28	1.517	28	0.320
RegCkt	95	48	2.600	48	0.400

C. Proposed method.

Thus, the proposed implementation method is summarized as follows.

1. We solve the initial circuit as shown in Fig. 7 by the DC analysis of SPICE, and obtain the initial point $\boldsymbol{x}^0$ and the constant term $\boldsymbol{f}(\boldsymbol{x}^0) + \tilde{\boldsymbol{f}}(\boldsymbol{x}^0)$ of the VGNH method. (Since the initial circuit is essentially a linear circuit, it can be solved by the DC analysis of SPICE.)
2. We perform the transient analysis of SPICE to the circuits shown in Figs. 6 and 8 starting from $(\boldsymbol{x}^0, 0)$ and trace the solution curve of (25a). If the solution curve reaches the $\lambda = 1$ hyperplane at $(\boldsymbol{x}^*, 1)$, then a solution $\boldsymbol{x}^*$ of (2) is obtained.

Since SPICE contains various efficient techniques such as sparse matrix techniques, variable-step variable-order implicit integration methods, and time-step control algorithms, a high-level VGNH method can be realized by the proposed method. Moreover, programming is not necessary and making the netlist of Figs. 6–8 is quite easy in the proposed method.

D. Examples.

We have applied the proposed implementation method to many practical circuits and have obtained good results. In this subsection, we show some examples. We used SPICE3f5 and the Sun Blade 2000.

Table IV shows the result of computation when we applied the VGNH method realized by i) our own program

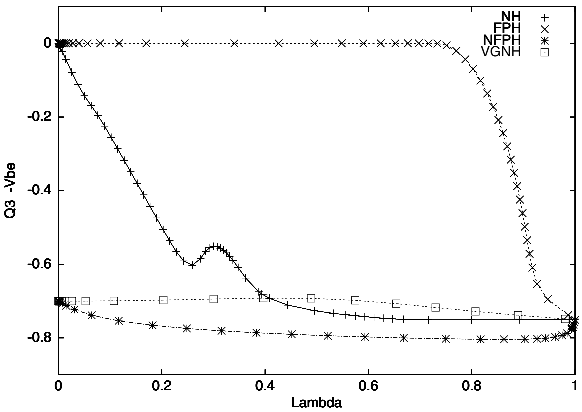

Fig. 9. Solution curves for HVRef (obtained by SPICE).

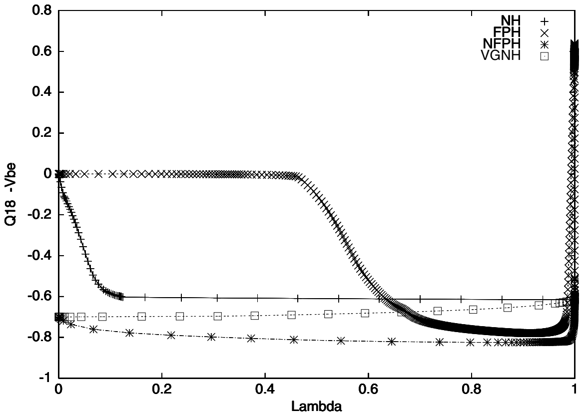

Fig. 10. Solution curves for μA741 (obtained by SPICE).

that was used in Section IV and ii) the proposed implementation method to the five transistor circuits discussed in Section IV. Since the number of steps changes by the parameters such as the initial step size and the maximum step size, we chose the parameters so that S becomes the same in the two approaches. From the table, it is seen that the proposed method is more efficient if the number of steps is the same. This is because SPICE contains various efficient techniques such as the sparse matrix techniques. Thus, we can implement an efficient VGNH method by using SPICE.

We also implemented the NH, FPH, and NFPH methods on SPICE using the similar idea. Figs. 9 and 10 show the solution curves when we applied these three methods and the VGNH method to HVRef and μA741, respectively. From these figures, it is seen that the number of marks of the VGNH method is the smallest, which implies that the solution curves are traced smoothly and efficiently in the proposed method.

VII. Conclusion

In this paper, an efficient and globally convergent homotopy method has been proposed for finding DC operating points of nonlinear circuits. Since the proposed method can use good initial points, and since it includes the concept of variable gain and does not include linear auxiliary functions, we can trace solution curves smoothly and efficiently. Furthermore, by using the method proposed in Section VI, we can implement a "sophisticated VGNH method with various efficient techniques" "easily" "without programming," "although we do not know the homotopy method well."

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