

# Relay Node Deployment Strategies in Heterogeneous Wireless Sensor Networks: Multiple-Hop Communication Case

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**Abstract** – While a lot of existing research attempts to extend the lifetime of a wireless sensor network (WSN) by designing energy efficient networking protocols, the impact of random device deployment on system lifetime is not stressed enough. Some research efforts have tried to optimize device deployment with respect to lifetime by assuming devices can be placed deliberately. However, the methodologies and solutions therein are not applicable to a randomly deployed large scale WSN. In this research, we propose three random deployment strategies for relay nodes in a heterogeneous WSN, namely, connectivity-oriented, lifetime-oriented and hybrid deployment. We investigate how a strategy can affect both connectivity and network lifetime of a multi-hop heterogeneous WSN, in which relay nodes transmit data to the base station via multi-hop relay. The performance of the three strategies is evaluated through simulations. The results of this research provide a viable solution to the problem of optimizing provisioning of a large scale heterogeneous WSN.

**Keywords:** *Wireless Sensor Network, Deployment, Biased Energy Consumption Rate, Lifetime, Connectivity*

## I. INTRODUCTION

Due to stringent energy constraints on small wireless devices, lifetime extension is one of the most critical technical concerns of WSN design. Many efforts have been made to improve the energy efficiency and extend the lifetime by designing energy efficient networking protocols. However, in a randomly deployed network, the significant influence of device deployment on the lifetime has been mostly overlooked.

Device deployment is a fundamental issue in WSN design. It determines many intrinsic properties of a WSN, such as coverage, connectivity, cost, and lifetime. It has been examined in terms of its effect on coverage and/or connectivity in [2-3]. However, the significance of deployment on lifetime in a randomly deployed network, in which the position of devices cannot be precisely known or controlled, is not addressed. A few research efforts have tried to optimize the device placement with respect to system lifetime [4-8]. However, they all assume the relay nodes (RNs), or high profile nodes, can be deliberately placed. Hence, the methodologies and solutions therein are not

applicable to the applications where deliberate placement is not feasible. The infeasibility usually occurs in two situations, one where the number of devices is very large, and the other where the application environment is not completely accessible. In these situations, well designed deployment density functions become viable approaches to optimizing the network provisioning.

To tackle the random deployment issue from a lifetime perspective, we must account for the biased energy consumption rate (BECR) phenomenon, described as follows. Consider a heterogeneous WSN composed of sensor nodes (SNs) and RNs. Traffic originates at SNs and is sent to the base station (BS) via RNs. Assume the traffic is uniformly generated over the sensing field and the initial energy is identical on every RN. In a single hop heterogeneous WSN, where RNs transmit data to the BS in one hop, such as in Fig.1(a), the RNs which are farther away from the BS will deplete energy faster than the RNs closer to the BS due to the larger transmission distance. When the dimension of the network is large enough, the nodes further from the BS become unusable while a large portion of energy is still left on those close to the BS. In contrast, in a multiple hop heterogeneous WSN, where RNs transmit data to the BS via multi-hop relay, such as Fig.1(b), RNs which are closer to the BS will consume energy faster than RNs farther away from the BS. The reason is because traffic is built up on RNs closer to the BS as it is relayed from far to near. As such, the RNs nearer to the BS will become unusable earlier than those far from the BS.

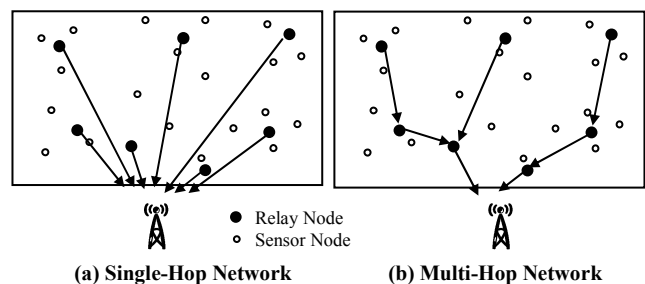


Fig. 1. BECR Problem in Heterogeneous WSNs

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The BECR phenomenon in a single-hop heterogeneous WSN has been studied in [1]. In this research, we examine the BECR phenomenon in a multi-hop heterogeneous WSN. Compared with the single-hop model, the multi-hop model also represents a practical application scenario, but it imposes more design challenges. As the RNs handle both local traffic and relay traffic, the computation of energy consumption has to consider the relay traffic buildup from far to near (to the BS). We propose three random deployment strategies for RNs, namely, connectivity-oriented deployment, lifetime-oriented deployment, and hybrid deployment. The performance of the three strategies is discussed in terms of lifetime and connectivity. Indeed, to our best knowledge, along with the research in [1], this is the first effort to optimize the random device deployment (by the density function) in order to extend the lifetime of a large scale heterogeneous WSN.

The remainder of this paper is organized as follows. In Section 2, related works are outlined. In Section 3, the system models are described. In Section 4, three random deployment strategies are proposed and the impact of deployment on connectivity and lifetime is discussed. In Section 5, the performance of the three strategies is evaluated and compared. In Section 6, we discuss the extensibility of our work. The paper is concluded in Section 7.

## II. RELATED WORK

Some research efforts have studied the deployment problem from the coverage and connectivity perspectives. In [2], Shakkottai et al. derive a necessary and sufficient condition for a random grid network to be covered and connected. The condition is presented as a scaling rule in terms of the number of nodes, the transmission radius and the probability of a node being functional. In [3], cost-minimizing sensor node placement strategies are proposed to provide complete coverage. Both works assume a homogenous network and do not consider the energy consumption on devices and network lifetime.

A few research efforts have examined the effect of deployment on system lifetime. In [4], Lee et al. first declare that in a heterogeneous network, lifetime, and quality and quantity of data processing can be enhanced by increasing the number of high profile devices, and coverage degree and coverage area can be augmented by using more low profile sensor nodes. They then study the tradeoff between these two aspects under a total cost constraint. They explore the optimal mixture of heterogeneous devices (number of each type) under both single-hop and multi-hop communication models. However, the device deployment is assumed to be uniform random for the low profile nodes and deliberately placed for high profile devices. The BECR phenomenon is not considered.

In [9], Mhatre et al. derive the most economical node deployment intensities and node energy that ensure a given

lifetime for a uniformly deployed WSN. The assumption that an aircraft periodically visits the sensing field, which equivalently makes the distances from cluster heads to the base station identical, void the BECR phenomenon, which is the core concern of this research.

The research in [5] aims to optimize the placement of the BS for maximizing the network lifetime when the positions of sensor nodes and application nodes (cluster heads) are given. The research in [6-7] is concerned with designing the optimal RN placement with the concerns of lifetime and connectivity. The research in [8] addresses the joint design problem of energy provisioning and relay node placement. The design problem is first formulated as a mixed-integer nonlinear programming problem, and a heuristic algorithm is derived to solve it. However, as the problem formulations and their solutions in these works depend on the exact positions and/or traffic of devices, the methodologies therein are not applicable to large-scale randomly deployed networks.

By avoiding the assumptions of device homogeneity, mobile data collectors, precise information of device positions and traffic, and deliberate placement, our research presents a practical and versatile approach to optimal provisioning of a large scale heterogeneous WSN.

## III. SYSTEM MODELS

### 3.1 Network Model

We assume a large-scale heterogeneous WSN on a sensing field  $A$  is composed of three types of devices, Sensor Nodes (SNs), Relay Nodes (RN) and a Base Station (BS). A SN senses the environment, generates data, and periodically transmits the data to an active RN<sup>1</sup>, which functions as a cluster head (CH), in a single hop. It has limited energy and a fixed transmission radius  $r_{SN}$ . It has no relaying function or at least traffic relaying is not a routine function of a SN for the following reasons. First, relaying traffic demands high intelligence, such as security and routing, which leads to higher device cost. Second, extra communication leads to faster energy dissipation. Thus a relay-providing SN will inevitably deplete its energy faster than its beneficiary. When the number of hops on a relaying path becomes large, this effect is aggravated due to traffic accumulation. Throughout the paper we assume  $N_{SN}$  SNs are randomly deployed in the network according to the uniform distribution and work simultaneously to meet the coverage requirement. This research can be extended to the case when SNs are redundant and work in shifts as long as the positions of on-duty SNs are randomly or evenly spread in the network. In this case  $N_{SN}$  represents the number of active SNs.

A RN is also energy constrained and has fixed transmission range  $r_{RN}$ , where typically  $r_{RN}$  is a few times larger than  $r_{SN}$ . A

<sup>1</sup> The research presented in this paper can be directly extended to the case when the traffic is not periodic, as long as the traffic is generated uniformly in the network.

RN works as a CH when active, which groups the SNs in its proximity into a cluster. It also coordinates and schedules the MAC layer access within its cluster so that the energy overhead, e.g., retransmissions due to collisions, is minimized. After receiving the data from SNs, it aggregates the traffic. The aggregation diminishes the redundant information from multiple nodes and reduces the network traffic. In the end, it transmits the aggregated data to the next hop active RN according to the routing algorithm running on these active RNs. The aggregated traffic won't be aggregated again while passing through other RNs. We assume the traffic is light compared with the available bandwidth, or the traffic is well scheduled so that there is no traffic congestion in the network.  $N_{RN}$  RNs are to be randomly deployed according to some strategy.

We assume one BS is fixed somewhere (e.g., the corner or center) in the sensing field. Without loss of generality, the position is marked as point (0, 0).

### 3.2 Energy Model

The energy spent by a SN for transmitting one packet to RNs is fixed as the transmission radius and packet lengths are fixed.

In one round of data collection, the energy spent by an active RN consists of two parts, i.e., the energy used for intra-cluster communication and data processing, denoted by  $E_{intra}$ , and the energy used for inter-cluster traffic relay, denoted by  $E_{inter}$ .

Consider a RN having  $n$  member SNs. The energy  $E_{intra}$  is composed of three parts, namely, the energy cost of receiving  $n$  packets of length  $l$ , denoted by  $E_{RX}(l, n)$ , the energy cost of transmitting the aggregated packet of length  $l_{AG}$  to its next hop RN or the BS over a distance  $r_{RN}$  (fixed transmission power/range), denoted by  $E_{TX}(l_{AG})$ , and the energy cost of aggregating  $n$  packets of length  $l$ , denoted by  $E_{AG}(l, n)$ . Adopting an energy model similar to that in [5], we have

$$E_{RX}(l, n) = nl\beta \quad (1)$$

$$E_{TX}(l_{AG}) = l_{AG}(\alpha_1 + \alpha_2 r_{RN}^m) \quad (2)$$

$$E_{AG}(l, n) = nl\gamma \quad (3)$$

where  $\beta$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $m$ , and  $\gamma$  are energy related parameters. Letting  $g$  be the aggregation ratio, the length of the aggregated packet from  $n$  packets is,

$$l_{AG}(l, n) = ngl \quad (4)$$

Replacing  $l_{AG}$  in (2) by (4), and adding (1), (2), and (3), we have

$$E_{intra} = c_1 nl \quad (5)$$

Where  $c_1 = (\beta + g\alpha_1 + \gamma + g\alpha_2 r_{RN}^m)$

On the other hand, the energy spent on inter-cluster relay  $E_{inter}$  consists of two parts, namely, the energy cost of

receiving packets of total length  $l_{Relay}$  and transmitting them (as they are) over the distance  $r_{RN}$ .

$$E_{inter} = c_2 l_{relay} \quad (6)$$

where  $c_2 = (\beta + \alpha_1 + \alpha_2 r_{RN}^m)$

Thus, the total energy spent by a RN in one round of data collection is,

$$E = c_1 nl + c_2 l_{relay} \quad (7)$$

### 3.3 Usability and Lifetime Model

The usability of a WSN is determined by both coverage and connectivity. Coverage has two aspects, i.e., coverage area and coverage degree [4]. In this research, they are ensured by the given SN deployment. Connectivity refers to how much of the generated data can ultimately arrive at the BS. It can be measured by the percentage of SNs that can connect to the BS via RNs. As such, coverage provided by SNs and connectivity provided by RNs ultimately determine the effective coverage. As RNs get drained of energy, the connectivity becomes gradually weaker and so does the effective coverage. This process is called *coverage aging* in [4]. As this research tries to extend the system lifetime from a connectivity perspective (topological lifetime in [5]), we define the system lifetime as the number of data collection rounds before the percentage of connected SNs is degraded to a given threshold  $q$ . We assume that SNs can function long enough (or have effective duty cycles) so that coverage is not hindered.

It is obvious that the percentage of SNs with connectivity in a newly deployed network should not be less than  $q$  in order that the network functions at all. We can achieve this by ensuring that the probability that any SN connects to at least one RN is not less than a value  $\sigma_0$ . The relationship between  $q$  and  $\sigma_0$  is presented in Appendix A. Throughout the paper, we use the fact that as long as the individual node connectivity probability is not less than  $\sigma_0$ , the overall connectivity satisfies  $q$ .

### 3.4 Routing Scheme

The research in this paper does not depend on the particular routing scheme used. We only assume that relay paths between RNs are always from far to near to the BS in order to avoid the unnecessarily long paths.

## IV. DEPLOYMENT STRATEGIES

In this section, we propose and examine deployment solutions for the following problem. Given a WSN as modeled in Section 3, how should one deploy a given number,  $N_{RN}$ , of RNs so that the network lifetime is maximized?

### 4.1 Connectivity-Oriented Deployment

Research in [9-14] has considered random deployment according to the uniform distribution. If  $N_{RN}$  RNs are deployed uniformly in a sensing field  $A$  of area  $|A|$ , for any SN, the probability that it can reach at least one RN in one hop is

$$p_R = 1 - \left(1 - \pi r_{SN}^2 / |A|\right)^{N_{RN}} \quad (8)$$

If a connection probability of  $\sigma_0$  for any SN is required,

i.e.  $p_R \geq \sigma_0$ , the minimum number of RNs is expressed as:

$$N_{RN}^{u\{\min\}} = \ln(1 - \sigma_0) / \ln(1 - \pi r_{SN}^2 / |A|) \quad (9)$$

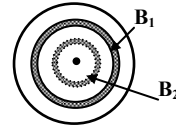
Compared with other random deployments, this strategy provides identical and maximal connectivity everywhere in the WSN. In other words, for a given connectivity requirement  $\sigma_0$ , this strategy will require the least number of RNs (illustrated in Section 5). We therefore refer to it as the Connectivity-Oriented Deployment Strategy. However, due to the BECR phenomenon discussed above, it suffers fundamentally from an energy efficiency perspective.

#### 4.2 Lifetime-Oriented Random Deployment

We also refer to this strategy as a weighted random deployment. The motivation is that the number of RNs deployed at different locations, and so the deployment density function, should be proportional to the expected energy dissipation rates at these locations.

Due to the add-up effects of the traffic relay and the randomness of the geometrical distribution of the RNs, deriving a provably optimal density function is a non-trivial task. Next, we present the derivation of a heuristic sub-optimal deployment density function. Indeed, the lifetime is increased up to more than 3 times by using the heuristic weighted deployment than the uniform deployment in our experimental setup. In the following, we first consider a circular sensing field of radius  $R$ , with the BS fixed at the center. We discuss how to extend the methodology to an arbitrary convex sensing field in Section 6. Points in the sensing field are given in polar coordinates in the following discussion.

The average deployment density in a given area should depend on two factors, namely the average total energy consumption rate in the area and the size of the area. The energy consumption rate of an area is the total energy consumed by RNs in the area per round of data collection. To overcome the BECR problem, the average density over an area should be proportional to the energy consumption rate and inversely proportional to the size of the area. For example, in Fig. 2, consider two arbitrary shells,  $B_1$  and  $B_2$ , with the BS at the center. The size of  $B_1$  is larger than that of  $B_2$ . Due to the BECR phenomenon, suppose that RNs in  $B_1$  and  $B_2$  have same energy consumption per round. Thus,  $B_2$  should have higher deployment density so that the expected numbers of RNs are the same in the two areas.



**Fig. 2.** A sensing site: the density function is proportional to the energy consumption rate, and is inversely proportional to the size of areas.

We therefore define the *Energy Consumption Intensity* (ECI) of an area as the ratio of the energy consumption rate of the area to the size of the area. For an arbitrary point  $(d, \theta)$  and a small positive value  $\mathcal{E}$ , we can form a disk of radius  $\mathcal{E}$  with  $(d, \theta)$  at the center. We define the ECI of position  $(d, \theta)$ , i.e.  $ECI(d, \theta)$  as the limit of the ECI of the disk as  $\mathcal{E}$  goes to 0. In fact, as the traffic is symmetric with respect to the BS,  $ECI(d, \theta)$  does not depend on  $\theta$ . The concept of  $ECI(d, \theta)$  is the basis for deriving the weighted random deployment density function. The principle is that the density function should be proportional to the ECI at any position.

To obtain the ECI, we next derive the amount of inter-cluster traffic and intra-cluster traffic at different parts of the network. We first define a parameter  $h_{RN}$  as  $h_{RN} = h * r_{RN}$ , where  $h$  is between 0 and 1. In Fig. 3, we construct the shell  $A_m$  of width  $h_{RN}$  lying between the two dotted circles in the sensing field. The area which is outside of  $A_m$  (farther from the BS) is referred to as  $A_{out}$ , and the area which is inside of  $A_m$  (closer to the BS) is referred to as  $A_{in}$ . Three types of traffic relay (between RNs) are of interest, first from  $A_{out}$  to  $A_m$ , second from  $A_m$  to  $A_m$ , and third from  $A_{out}$  to  $A_{in}$  directly. When  $h=1$ , the direct relay from  $A_{out}$  to  $A_{in}$  does not exist and some relay happens from RNs in  $A_m$  to other RNs in  $A_m$ . As  $h$  becomes smaller (the width of the shell decreases), the relay from  $A_{out}$  to  $A_{in}$  directly becomes more common and so more traffic from RNs in  $A_{out}$  will not be relayed by RNs in  $A_m$ . At the same time, less traffic from RNs in  $A_m$  will be relayed to other RNs in  $A_m$ . By empirically choosing the value of  $h$  appropriately, the amount of traffic relayed from  $A_{out}$  to  $A_{in}$  directly and the amount of traffic relayed between RNs inside the shell  $A_m$  are largely cancelled out by each other. In such a case, we can approximate the volume of inter-cluster traffic relayed by RNs in the shell  $A_m$  by all traffic generated by RNs in  $A_{out}$ . We will explore the optimal value of  $h$  in Section 5. Also, the average intra-cluster traffic volume handled by the RNs in any sub-area is proportional to the size of the sub-area under consideration. That is, the intra-cluster traffic handled by RNs in the shell  $A_m$  is the traffic originated by SNs located in the same shell. Following the same logic, the relay traffic transmitted from  $A_{out}$  to  $A_m$  is the sum of the aggregated traffic generated by SNs in  $A_{out}$ . The approximations on the inter-cluster and intra-cluster traffic volume of the shell  $A_m$  are the basis for the following derivation.

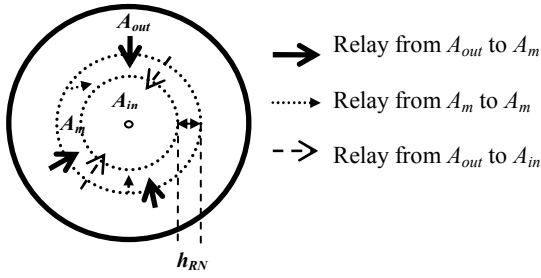


Fig. 3. The physical meaning of the effective radius of RNs

With  $h_{RN}$ , we partition a sensing field with radius  $R$  into three areas as shown in Fig.4. The part which is surrounded by the inner broken circle of radius  $r_{RN}$  is the first area, denoted by  $A_1$ . In this area, a RN is able to transmit to the BS in one hop. The shell between the two broken circles of radius of  $R-h_{RN}$  and  $r_{RN}$  respectively, is the second area, denoted by  $A_2$ . In this area, traffic is relayed from far to near. The remaining part, which is between the bounding solid circle of radius  $R$  and the broken circle of radius  $R-h_{RN}$ , is the third area, denoted by  $A_3$ . The inter-cluster relay traffic is negligible in area  $A_3$ . The three areas are defined as,

$$A_1 = \{(d, \theta) \mid 0 \leq d \leq \min(r_{RN}, R), 0 \leq \theta \leq 2\pi\} \quad (11)$$

$$A_2 = \begin{cases} \{(d, \theta) \mid r_{RN} < d \leq R - h_{RN}, 0 \leq \theta \leq 2\pi\}; & R > r_{RN} + h_{RN} \\ \phi, & \text{otherwise} \end{cases} \quad (12)$$

$$A_3 = \begin{cases} \{(d, \theta) \mid \max(r_{RN}, R - h_{RN}) < d \leq R, 0 \leq \theta \leq 2\pi\}; & R > r_{RN} \\ \phi, & \text{otherwise} \end{cases} \quad (13)$$

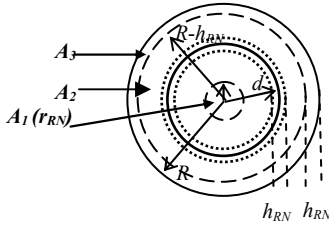


Fig. 4. Partition of a Sensing Site

Note that, if  $R \leq r_{RN}$ ,  $A_2$  and  $A_3$  shrink to null sets, and if  $r_{RN} < R \leq h_{RN} + r_{RN}$ ,  $A_2$  shrinks to a null set. Without loss of generality, we consider the case where  $R > h_{RN} + r_{RN}$ . The other two cases are easily addressed following the same line of logic.

In  $A_1$ , the expected number of SNs is  $N_{SN} r_{RN}^2 / R^2$ . Substituting for  $n$  in (5), the expected total energy spent on intra-cluster communication by all RNs in  $A_1$  is,

$$E_{\text{intra}}^{(1)} = c_1 N_{SN} l r_{RN}^2 / R^2 \quad (14)$$

All traffic generated by SNs outside of  $A_1$  must be relayed by a RN in  $A_1$  to reach the BS. The expected traffic relayed by RNs in  $A_1$  is  $g N_{SN} l (R^2 - r_{RN}^2) / R^2$ . Substituting for  $l_{\text{relay}}$  in (6), the expected total energy spent on inter cluster relay by RNs in  $A_1$  is,

$$E_{\text{inter}}^{(1)} = c_2 g N_{SN} l (R^2 - r_{RN}^2) / R^2 \quad (15)$$

We make the approximation that the ECI at any position  $(d, \theta)$  in  $A_1$  is the same and it can be approximated by

$$ECI^{(1)}(d, \theta) = \left( \frac{E_{\text{intra}}^{(1)} + E_{\text{inter}}^{(1)}}{\pi r_{RN}^2} \right) \quad (16)$$

$$= \frac{N_{SN} l}{\pi R^2} \left( c_1 + c_2 g \left( \frac{R^2}{r_{RN}^2} - 1 \right) \right)$$

The integral of  $ECI^{(1)}(d, \theta)$  over  $A_1$ , denoted by  $J^{(1)}$  is

$$J^{(1)} = \frac{N_{SN} l}{R^2} (c_1 r_{RN}^2 + c_2 g (R^2 - r_{RN}^2)) \quad (17)$$

In  $A_2$ , the ECI at different positions might be largely differentiated, as RNs at different positions relay different amounts of traffic. We propose to approximate the ECI at point  $(d, \theta)$  by the ECI of the shell between two dotted circles of radius  $(d - h_{RN}/2)$  and  $(d + h_{RN}/2)$  (see Fig.4), which is calculated as the sum of the energy consumption for intra-cluster communication,  $E_{\text{intra}}^{(2)}(d)$  and the energy consumption for the inter-cluster relay,  $E_{\text{inter}}^{(2)}(d)$ , by RNs in the shell, divided by the size of the shell, i.e.,

$$ECI^{(2)}(d, \theta) = \left( \frac{E_{\text{intra}}^{(2)}(d) + E_{\text{inter}}^{(2)}(d)}{\pi \left( (d + h_{RN}/2)^2 - (d - h_{RN}/2)^2 \right)} \right) \quad (18)$$

Similar to (14), the energy consumption for intra-cluster traffic in the shell for each round of data collection is approximated as,

$$E_{\text{intra}}^{(2)}(d) = \frac{2c_1 N_{SN} l d h_{RN}}{R^2} \quad (19)$$

The energy consumption for inter-cluster traffic in the shell for each round of data collection can be approximated by,

$$E_{\text{inter}}^{(2)}(d) = c_2 g N_{SN} l \left( \frac{R^2 - (d + h_{RN}/2)^2}{R^2} \right) \quad (20)$$

Plugging (19) and (20) into (18), we have,

$$ECI^{(2)}(d, \theta) = \frac{N_{SN} l}{\pi R^2} \left[ c_1 + \frac{c_2 g}{2d h_{RN}} \left( R^2 - \left( d + \frac{h_{RN}}{2} \right)^2 \right) \right] \quad (21)$$

The integral of  $ECI^{(2)}(d, \theta)$  over  $A_2$ , denoted by  $J^{(2)}$  is

$$J^{(2)} = \frac{N_{SN} l}{R^2} \left[ c_1 \left( (R - h_{RN})^2 - r_{RN}^2 \right) + c_2 g \left( R^2 (R - h_{RN} - r_{RN}) + \frac{(2r_{RN} + h_{RN})^3 - (2R - h_{RN})^3}{24} \right) \right] \quad (22)$$

For  $A_3$ , the traffic of inter-cluster relaying is negligible. Similar to  $A_1$ , the ECI at any position  $(d, \theta)$  in  $A_3$  is,

$$ECI^{(3)}(d, \theta) = \frac{c_1 N_{SN} l}{\pi R^2} \quad (23)$$

The integral of  $ECI^{(3)}(d, \theta)$  over  $A_3$ , denoted by  $J^{(3)}$  is

$$J^{(3)} = \frac{c_1 N_{SN} l (2R - h_{RN}) h_{RN}}{R^2} \quad (24)$$

Let  $J = J^{(1)} + J^{(2)} + J^{(3)}$ . We propose the density function for the three areas as follows.

$$f(d, \theta) = \begin{cases} ECI^{(1)}(d, \theta) / J, & \text{if } (d, \theta) \in A_1 \\ ECI^{(2)}(d, \theta) / J, & \text{if } (d, \theta) \in A_2 \\ ECI^{(3)}(d, \theta) / J & \text{if } (d, \theta) \in A_3 \end{cases} \quad (25)$$

In the following, we discuss the properties of the deployment density in (25) in terms of connectivity. If  $N_{RN}$  RNs are deployed according to the density function (25), the probability that a SN, at point  $(d, \theta)$ , can reach one or more RNs in one hop is,

$$p_R(d, \theta) = 1 - \left( 1 - \int_{O(d, \theta)} f(u, v) u du dv \right)^{N_{RN}} \quad (26)$$

where  $O(d, \theta)$  is a disk centered at  $(d, \theta)$  with radius  $r_{SN}$ . If  $r_{SN}$  is small (compared with the overall field), the probability can be approximated by

$$p_R(d, \theta) = 1 - \left( 1 - \pi r_{SN}^2 f(d, \theta) \right)^{N_{RN}} \quad (27)$$

For a SN whose transmission disk is in  $A_1$ , the connectivity probability is,

$$p_R^{(1)}(d, \theta) = 1 - \left( 1 - \pi r_{SN}^2 ECI^{(1)}(d, \theta) / J \right)^{N_{RN}} \quad (28)$$

If the connectivity probability  $\sigma_0$  is required, letting

$$p_R^{(1)}(d, \theta) = \sigma_0, \text{ and solving for } N_{RN}, \text{ we have,} \quad (29)$$

$$N_{RN}^{w\{\min 1\}} = \ln(1 - \sigma_0) / \ln(1 - \pi r_{SN}^2 ECI^{(1)}(d, \theta) / J)$$

That is, if  $N_{RN}$  is equal to or larger than  $N_{RN}^{w\{\min 1\}}$ , the deployment according to (25) will be able to meet the connectivity requirement in  $A_1$ .

Similarly, in  $A_2$ , the connectivity probability of a SN is

$$p_R^{(2)}(d, \theta) = 1 - \left( 1 - \pi r_{SN}^2 ECI^{(2)}(d, \theta) / J \right)^{N_{RN}} \quad (30)$$

Now  $p_R^{(2)}(d, \theta)$  is a decreasing function in  $[r_{RN}, R - h_{RN}]$ . Thus, a SN at distance  $R - h_{RN}$  has the least connectivity probability  $p_R^{(2)}(R - h_{RN}, \theta)$ . Letting  $p_R^{(2)}(R - h_{RN}, \theta)$

$$= \sigma_0, \text{ solving for } N_{RN} \text{ we have,} \quad (31)$$

$$N_{RN}^{w\{\min 2\}} = \ln(1 - \sigma_0) / \ln(1 - \pi r_{SN}^2 ECI^{(2)}(R - h_{RN}, \theta) / J)$$

That is, if  $N_{RN}$  is equal to or larger than  $N_{RN}^{w\{\min 2\}}$ , the deployment according to (25) will be able to meet the connectivity requirement everywhere in  $A_2$ .

In  $A_2$ , SNs at distance  $r_{RN}$  from the BS have the highest connectivity probability. Setting  $p_R^{(2)}(r_{RN}, \theta) = \sigma_0$ , and solving for  $N_{RN}$ , we have,

$$N_{RN}^{w\{\min 2-\}} = \ln(1 - \sigma_0) / \ln(1 - \pi r_{SN}^2 ECI^{(2)}(r_{RN}, \theta) / J) \quad (32)$$

That is, if  $N_{RN}$  is less than  $N_{RN}^{w\{\min 2-\}}$ , the deployment according to (25) will not be able to meet the connectivity requirement anywhere in  $A_2$ .

If  $N_{RN}^{w\{\min 2-\}} \leq N_{RN} < N_{RN}^{w\{\min 2\}}$ , the deployment according to (25) will be able to meet the connectivity requirement partially in  $A_2$ , but not everywhere. In this case, letting  $p_R^{(2)}(d, \theta) = \sigma_0$ , we can solve for  $d$  using Newton's method as  $p_R^{(2)}(d, \theta)$  is a decreasing function of  $d$  on  $[r_{RN}, R - h_{RN}]$ . Let the solution be  $d_0$ . It defines a cutoff distance inside the sensing area  $A_2$ . We define the region  $B$  as

$$B = \{(d, \theta) \mid d_0 < d \leq R - h_{RN}\} \quad (33)$$

In this region, the connectivity probability of a SN is less than  $\sigma_0$ ; at positions in  $A_2$  other than  $B$ , a SN has connectivity probability higher than  $\sigma_0$ . The cutoff circle and region  $B$  (the tinted area) is illustrated in Fig. 5.

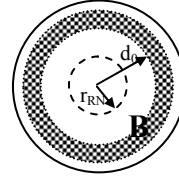


Fig. 5. The connectivity in  $B$  is not satisfied

In  $A_3$ , the connectivity probability of a SN is at least

$$p_R^{(3)}(d, \theta) = 1 - \left( 1 - \pi r_{SN}^2 ECI^{(3)}(d, \theta) / J \right)^{N_{RN}} \quad (34)$$

Letting  $p_R^{(3)}(d, \theta) = \sigma_0$ , and solving for  $N_{RN}$ , we have

$$N_{RN}^{w\{\min 3\}} = \ln(1 - \sigma_0) / \ln(1 - \pi r_{SN}^2 ECI^{(3)}(d, \theta) / J) \quad (35)$$

That is, if  $N_{RN}$  is larger than  $N_{RN}^{w\{\min 3\}}$ , the deployment according to (25) will be able to meet the connectivity requirement in  $A_3$ .

Define  $N_{RN}^{w\{\min\}} = \max\{N_{RN}^{w\{\min 1\}}, N_{RN}^{w\{\min 2\}}, N_{RN}^{w\{\min 3\}}\}$ . So, if the number of RNs,  $N_{RN}$ , is equal to or greater than  $N_{RN}^{w\{\min\}}$ , the connectivity of SNs is satisfied everywhere in the network.

### 4.3 Hybrid Deployment

The weighted random deployment of RNs according to the density function (25) can counteract the BECR phenomenon. However, this benefit will be materialized only if the connectivity of SNs is satisfied in the network. If the number of given RNs,  $N_{RN}$ , is less than  $N_{RN}^{w\{\min\}}$ , the number of SNs without connectivity may be too high for a network to function at all.

The objective of the hybrid deployment is to optimize RN deployment by balancing the concerns of connectivity and

lifetime extension. If  $N_{RN} < N_{RN}^{u\{\min\}}$ , there is no way to guarantee the connectivity at the first place. If  $N_{RN} \geq N_{RN}^{w\{\min\}}$ , the weighted random deployment as defined by (25) can provide the satisfying connectivity. If  $N_{RN}^{u\{\min\}} \leq N_{RN} < N_{RN}^{w\{\min\}}$ , the weighted random deployment alone will not be able to satisfy the connectivity. In this case, the hybrid deployment tries to maximize the system lifetime while satisfying the connectivity requirement. To this end, the hybrid deployment is designed in two steps. Firstly, we design the deployment of  $N_{RN}^l$  RNs in a weighted random manner as defined by (25). Since the connectivity of SNs is not satisfied outside the circle of radius  $d_0$ , in the second step we arrange the deployment of  $N_{RN}^c$  RNs so as to compensate for the weaker connectivity in this area. The total number of RNs deployed in the two steps should be equal to the given number  $N_{RN}$ . We next study how  $N_{RN}$  should be optimally split between  $N_{RN}^l$  and  $N_{RN}^c$ .

Allocation of RNs for the two steps is a constrained optimization problem. As  $N_{RN}^l$  increases,  $N_{RN}^c$  has to be decreased. However, if  $N_{RN}^c$  is too small, the connectivity of the sparse area of the network is at risk. In the following, we consider an arbitrary  $n_{RN}^l < N_{RN}$  for the first step. To satisfy the connectivity in weak areas, we derive the number of RNs needed in the second step  $n_{RN}^c$  (enhance connectivity in weak areas) as a function of  $n_{RN}^l$ . By summing  $n_{RN}^l$  and  $n_{RN}^c$  (function of  $n_{RN}^l$ ), we obtain the total number of RNs  $n_{RN}$  as a function of  $n_{RN}^l$ . We prove that  $n_{RN}$  is a non-decreasing function of  $n_{RN}^l$ . Thus, we can easily solve for  $N_{RN}^l$  for given  $N_{RN}$ .

Assume that  $n_{RN}^l$  RNs have been deployed according to (25).

We can calculate the number of RNs  $n_{RN}^c$  as the sum of RNs needed in three parts respectively.

In the set  $A_1$ , the number of RNs needed is

$$n_{RN}^{c1} = \max(0, \ln(1 - \sigma_0) / \ln(1 - r_{SN}^2 / r_{RN}^2) - n_{RN}^l J^{(1)} / J) \quad (36)$$

Similarly, in  $A_3$ , the number needed is

$$n_{RN}^{c3} = \max(0, \ln(1 - \sigma_0) / \ln(1 - r_{SN}^2 / (2Rh_{RN} - h_{RN}^2)) - n_{RN}^l J^{(3)} / J) \quad (37)$$

For the set  $A_2$ , we examine the compensation deployment in two cases. The 1<sup>st</sup> case is  $N_{RN}^{w\{\min 2-\}} \leq N_{RN} < N_{RN}^{w\{\min 2\}}$ ; and the 2<sup>nd</sup> case is  $N_{RN} < N_{RN}^{w\{\min 2-\}}$ . In the 1<sup>st</sup> case, the connectivity is partially satisfied in  $A_2$ . We define the RN density at a position  $(d, \theta)$  as the product of the number of RNs deployed and the density function  $f(d, \theta)$ . To make the

connectivity in the set  $B$  meet the minimum requirement, the RN density in  $B$  should be leveled up to the RN density level of points  $(d_0, \theta)$  on the boundary of  $B$ . The number of RNs needed in the second step is

$$n_{RN}^{c2} = \left[ \int_B (n_{RN}^l \cdot f(d_0, \theta) - n_{RN}^l \cdot f(x, \theta)) dx d\theta \right] \quad (38)$$

Plugging (18) and (25) into (38), we have

$$n_{RN}^{c2} = \left[ \frac{c_2 g_{RN}^l N_{SN}^l \left( \frac{((R - h_{RN})^2 - d_0^2)(R^2 - (d_0 + h_{RN}/2)^2)}{2d_0} \right) - c_2 g_{RN}^l N_{SN}^l \left( R^2 (R - h_{RN} - d_0) + \frac{(2d_0 + h_{RN})^3 - (2R - h_{RN})^3}{24} \right) \right] \quad (39-1)$$

In the 2<sup>nd</sup> case, the connectivity is not satisfied anywhere in  $A_2$ . The number of RNs in the second step is then

$$n_{RN}^{c2} = \left[ (N_{RN}^{u\{\min\}} - n_{RN}^l) \frac{((R - h_{RN})^2 - r_{RN}^2)}{R^2} \right] \quad (39-2)$$

Summing  $n_{RN}^l$ ,  $n_{RN}^{c1}$ , the total number of RNs deployed is

$$n_{RN} = n_{RN}^l + n_{RN}^{c1} + n_{RN}^{c2} + n_{RN}^{c3} \quad (40)$$

**Lemma:**  $n_{RN}$  is a non-decreasing function of  $n_{RN}^l$ .

Proof: Appendix B.

For the compensation deployment, the number of RNs for each part can be calculated using formulas (36) (37) and (39). The density function for areas  $A_1$  and  $A_3$  is uniform. The density function for the region  $B$  in  $A_2$  in the 1<sup>st</sup> case is

$$g(d, \theta) = \frac{f(d_0, \theta) - f(d, \theta)}{\int_0^{2\pi} \int_{d_0}^{R-h_{RN}} (f(d_0, \theta) - f(u, v)) u du dv} \quad (41-1)$$

In the 2<sup>nd</sup> case, the density function for all of  $A_2$  is

$$g(d, \theta) = \frac{N_{RN}^{u\{\min\}} / \pi R^2 - n_{RN}^l f(d, \theta)}{\int_0^{2\pi} \int_{d_0}^{R-h_{RN}} (N_{RN}^{u\{\min\}} / \pi R^2 - n_{RN}^l f(u, v)) u du dv} \quad (41-2)$$

After the second step, the RN density becomes uniform everywhere in  $B$  and the connectivity is satisfied everywhere.

## V. PERFORMANCE EVALUATION

In this section, the three proposed deployment strategies are evaluated in various design scenarios using simulations. The simulation implements a clustering scheme as follow.

### 5.1 Clustering Scheme

As RNs are densely deployed, energy is wasted if all of them work simultaneously. A clustering algorithm is used to select CHs from redundant RNs, so that some RNs can connect all SNs while other RNs go to sleep. Most existing clustering algorithms are designed for homogenous networks and they assign the role of CH to identical nodes in rotation [10-14]. Such schemes cannot be directly applied or extended to the case of heterogeneous networks.

To conduct a convincing performance evaluation and a fair comparison of the deployment strategies, we propose a simple and effective idealized clustering scheme for heterogeneous WSNs. Assuming every RN sets up a neighboring SN table upon initialization, the operation of our scheme is briefly described as follows:

- a) A RN is elected as a CH if it covers the most uncovered SNs, and it broadcasts the ADVERTISEMENT message to its neighboring RNs.
- b) A RN goes to sleep if all of its neighboring SNs are covered by active CHs (known from the ADVERTISEMENT messages).
- c) A CH keeps functioning until its energy is exhausted. In this case, the clustering scheme is locally invoked to select other CHs. The election gives preference to the RNs which cover the most uncovered SNs.
- d) Depleted RNs will not be involved in any further operations.

The scheme has the following desired properties. Firstly, it ensures that each SN is able to reach a CH, unless all neighboring RNs are out of energy. Secondly, the clustering scheme tries to minimize the number of CHs. Thirdly, the CH duty cycle is rotated in an on-demand manner. Only a CH which is going out of energy needs to invoke a local CH selection procedure.

After CHs have been locally selected in the network to connect SNs directly, CHs execute the Bellman-Ford algorithm to set up the paths to the BS. If two neighboring RNs have the shortest paths (to the BS) of the same hop, the one with less traffic is chosen.

### 5.2 Performance Metrics and Simulation Setup

Two metrics are used to measure the performance. The first one is the utilization of energy in the system, i.e., the ratio of the total consumed energy of RNs to the total initial energy. The other metric, denoted by Normalized DCR, is the number of data collection rounds normalized by the initial energy of a RN (the unit is Joule) before the network lifetime expires. In each round, every SN transmits one packet to its associated CH.

We simulate a WSN of 10,000 SNs on a disk sensing field with radius 500m, in which the BS is located at the center. The parameters used in the simulations are listed in Table 1. Therein,  $\sigma_0$  is calculated according to Appendix A to ensure (with probability greater than 0.9999) that the ratio of total connected SNs in an initial deployment is not less than  $q$ . All experimental results presented are the average of 30 runs.

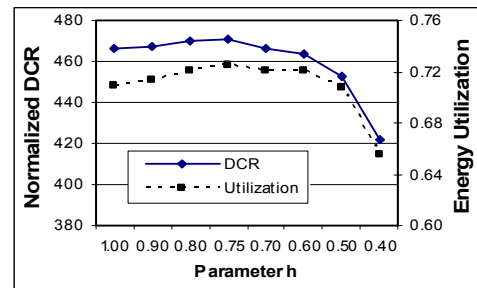
### 5.3 The Impact of Parameter $h$

The derivation in Section 4.2 depends on an ad hoc parameter  $h$ . We first investigate how this design parameter affects the performance of the lifetime oriented deployment strategy and find out the best value for the simulation setup above. We

conduct the lifetime oriented deployment strategy using different values of  $h$  from 0.4 to 1.0. To make the comparison fair and effective, the number of RNs to be deployed is set to 2500, which is greater than  $N_{RN}^{w(\min)}$  for all cases ( $N_{RN}^{w(\min)}$  is a function of  $h$ ). In other words, with 2500 RNs deployed by the weighted density function, the connectivity requirement is satisfied for all cases. The results are presented in Fig. 6.

**Table 1.** The parameters of the simulated WSN

$\alpha_1$	50e-9 (J/bit)	$m$	2
$\alpha_2$	10e-12 (J/bit/m <sup>2</sup> )	$g$	0.2
$\beta$	50e-9 (J/bit)	$\gamma$	1e-12 (J/bit)
$N_{SN}$	10,000	$R$	500 (m)
$r_{RN}$	90 (m)	$r_{SN}$	30 (m)
$q$	0.8	$\sigma_0$	0.84
$l$	2000 (bits)		



**Fig. 6.** Comparison of the lifetime oriented deployments with different  $h$  by energy utilization and DCR

The results for both the energy utilization and the system lifetime (Normalized DCR) indicate the same trend. First of all, the weighted random deployment performs the best at  $h=0.75$  for the given setup. Generally speaking, the performance varies slightly when  $h$  is between 0.6 and 1. From 0.75 to 1, the performance of the weighted random deployment degrades gradually as  $h$  rises. When  $h < 0.75$ , the performance degrades as  $h$  decreases and the drop is accelerated as  $h < 0.5$ . It is safe to reason that the drop will continue as  $h$  decreases further. Therefore, in the experiments which follow, we always use  $h=0.75$  for the weighted random strategy and corresponding hybrid strategy.

### 5.4 Comparison of Deployment Strategies

In this section, we explore and compare the performance of each of the strategies from Section 4. We have determined an optimal value for  $h$  equal to 0.75. Some key properties of the connectivity oriented deployment and the weighted deployment (when  $h=0.75$ ) are given in Table 2. The three strategies are experimented on the same network while  $N_{RN}$  varies from 509 to 3000. (According to (9), if the number of

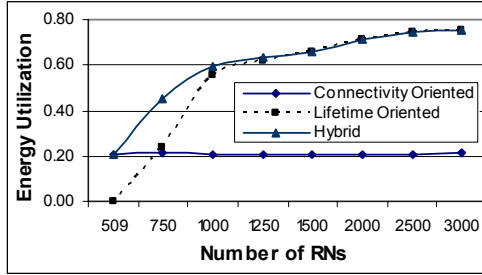


RNs is less than 509, none of the strategies can provide a functioning network upon startup with high probability.)

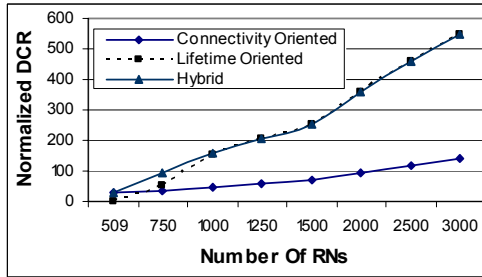
**Table 2.** Key properties of deployment strategies.

$N_{RN}^{u\{\min\}}$	509	$N_{RN}^{w\{\min 1\}}$	98
$N_{RN}^{w\{\min 2\}}$	1181	$N_{RN}^{w\{\min 3\}}$	1495

Fig.7 and Fig.8 present the results of the energy utilization and the Normalized DCR by using the three strategies.



**Fig. 7.** Comparison of three deployment strategies by energy utilization



**Fig. 8.** Comparison of three deployment strategies by DCR

For the connectivity oriented deployment, the energy utilization is almost unchanged at around 21% as the number of RNs increases from 509 to 3000. The energy wastage due to the BECR problem is clearly exemplified. On the other hand, the Normalized DCR increases approximately linearly as the number of RNs increases.

In contrast, the lifetime oriented deployment exhibits much better performance when  $N_{RN} > 750$ . The energy utilization increases rapidly from 24% when  $N_{RN} = 750$  to 66% when  $N_{RN} = 1500$ . The rate of rise becomes milder when  $N_{RN} > 1500$  and reaches 75% when  $N_{RN} = 3000$ . The reason is that the weighted density function reflects the energy consumption at different locations, not only from the local traffic, but also from the traffic relayed from far to near. Its benefits are better realized as  $N_{RN}$  is larger and the connectivity is provided with high probability. As a result, the Normalized DCR increases much faster than the connectivity oriented deployment as  $N_{RN}$  gets larger. When  $N_{RN} = 3000$ , the utilization of the lifetime oriented deployment is more than three times of that of the connectivity oriented deployment. It is similar for the Normalized DCR. When  $N_{RN} = 509$ , the deployment according

to the weighted random density function cannot satisfy the connectivity requirement, and the initial network is unusable.

The hybrid deployment is the best deployment strategy of the three. When  $N_{RN} = 509$ , the hybrid deployment is equal to the connectivity oriented deployment as the number of RNs allocated for the first step is 0. All RNs are used to meet the minimal connectivity (Section 4.3). When  $509 < N_{RN} < 1500$ , the hybrid deployment provides better performance than the lifetime oriented deployment since it reconciles the needs of lifetime extension with the connectivity. The advantage becomes less significant as  $N_{RN}$  increases due to the fact that the connectivity issue becomes a less serious problem as  $N_{RN}$  approaches  $N_{CH}^{w\{\min\}} = 1495$ . When  $N_{RN} > N_{CH}^{w\{\min\}}$ , there is no difference between the hybrid deployment and lifetime oriented deployment.

The general trend of the weighted density function is that the farther a position is away from the BS, the less density it is awarded. We wonder if there exist some decreasing functions in a simple form can provide a similar or better performance at all. We investigate this thought by considering two simple forms of decreasing functions as optional deployment density functions. We conduct experiments on them and compare the results with those of the weighted density function.

The first one is a quadratic density function. Consider a shell of width  $\mathcal{E}$  (a small value) at distance  $d$ . A quick estimate of the traffic passing by the RNs in the shell is approximately equal to the traffic generated from SNs farther than  $d$  (from the BS). The expected number of SNs whose distance from the BS is equal to or greater than  $d$  is proportional to  $(R^2 - d^2)$ , and so is the traffic volume passing by the shell. We therefore propose a quadratic density function given by<sup>2</sup>

$$f(d, \theta) = \frac{2(R^2 - d^2)}{\pi R^4} \quad (42)$$

Another simple function we consider is the linear density function given by<sup>2</sup>

$$f(d, \theta) = \frac{3(R - d)}{\pi R^3} \quad (43)$$

We implement the deployment according to the density functions (25), (42) and (43) with 2000, 2500 and 3000 RNs. The density functions are first plotted and compared in Fig.9. The experimental results are presented in Fig.10 and Fig.11.

<sup>2</sup> The density functions are not arbitrary. The integral of the quadratic density function and linear density function is 1.

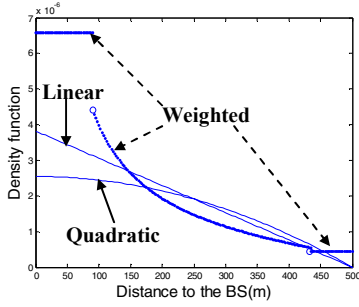


Fig. 9. Three deployment density functions

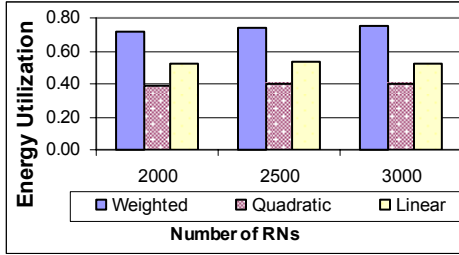


Fig. 10. Comparison of three density functions by energy utilization

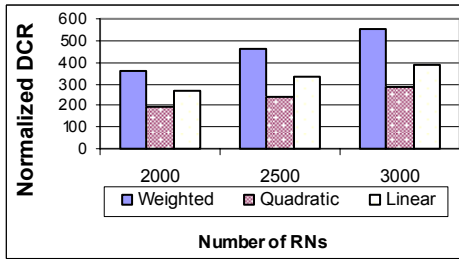


Fig. 11. Comparison of three density functions by DCR

Generally speaking, the weighted density function given by (25) performs the best of the three functions in all cases. Both the linear density function and quadratic function overcome the BECR problem to some degree. However, the performance of the linear function performs always better than the quadratic function. Actually, the formula (21), which determines the deployment in  $A_2$  (from radius 90m to 432.5 m) is composed of a linear function of  $d$  and a inverse function of  $d$ . It partially explains the advantage of the linear function over the quadratic function.

## VI. EXTENSIBILITY

The derivation in Sections 4.2 and 4.3 assumes a sensing field of circular shape. However, the method and the derivations can be extended to the case where the sensing field is of arbitrary convex shape and the BS is inside or outside the sensing field. For example, in Fig. 12, SNs are uniformly deployed in a sensing field  $S$ , represented by the solid irregular curve and the BS is outside of  $S$ . In such a case, draw two lines (broken lines in Fig. 12) from the BS tangent to the boundary of  $S$ . Thus, we can determine a RN deployment density function for the area surrounded by the irregular curve and the tangent lines, denoted by  $S'$ .

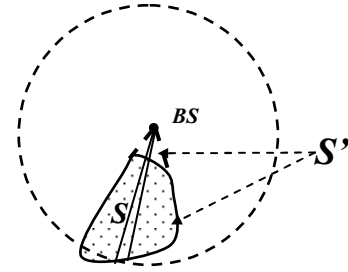


Fig. 12. Irregular sensing site  $S$  and the BS is out of  $S$ . RNs are to be deployed in  $S'$ .

The deployment design starts by cutting  $S'$  into slices with the center at the BS so that the difference in distances from the points on the edge of each slice to the BS are negligible. We then try to derive the ECI of each position in the slice. The overall deployment density function is the ECI divided by the integral of ECI over  $S'$ . Conceptually, it is equivalent to (25).

In order to find the ECI function for a slice  $S_i$ , we imagine expanding the slice into a disk of the same radius, as denoted by the dotted circle in Fig. 12. The ECI function at different positions can be derived as in (11)-(25), except that if the position is out of the sensing field, the intra-cluster traffic is 0. The similar derivation is validated by a key feature of  $ECI(d, \theta)$ , i.e., it is a function of distance only. The hybrid deployment can be further derived once the weighted density function is determined. The general procedure is finding the weakly connected areas and deploying some RNs to compensate for them. Lemma 1 is generally applicable. The detailed derivation of both deployment strategies is not given here due to the limited space.

## VII. CONCLUSION

Device deployment is a fundamental issue in WSNs. The number and positions of devices determine the usability of a system in terms of coverage, connectivity, lifetime, cost, etc. In this paper, we study the influence of random device deployment on connectivity and lifetime in a large-scale heterogeneous WSN. In particular, we examine the biased energy consumption rate problem in a multi-hop WSN. Based on it, we propose three deployment strategies, namely, connectivity-oriented, lifetime-oriented, and hybrid (balancing connectivity and lifetime goals). The performance of the strategies is further investigated by simulations. The hybrid deployment reconciles the concerns of lifetime with connectivity, and we conclude it is a preferred solution. This paper provides a guideline for deployment of typical large scale heterogeneous WSNs.

## Appendix A

In this appendix, we discuss the relationship of  $q$  and  $\sigma_0$ . If the connection probability of any individual SN is  $\sigma$ , the probability that  $x$  out of  $N_{SN}$  SNs are connected has the binomial distribution with parameter  $(N_{SN}, \sigma)$ . When  $N_{SN} \sigma$

and  $N_{SN}(1-\sigma)$  are big enough, this binomial distribution can be approximated by the normal distribution with parameters  $(N_{SN}\sigma, \sqrt{N_{SN}\sigma(1-\sigma)})$ . If we want the deployment to satisfy the functionality threshold  $q$  with a high probability (say 0.9999 and above), the minimum connection probability of any individual SN, denoted by  $\sigma_0$ , can be obtained as a function of  $q$ .

## Appendix B

**Lemma:**  $n_{RN}$  is an non-decreasing function of  $n_{RN}^l$ .

Pick two integers  $n_{RN}^{l1}$  and  $n_{RN}^{l2}$  where  $n_{RN}^{l1} > n_{RN}^{l2}$ . Consider two deployments in which  $n_{RN}^{l1}$  and  $n_{RN}^{l2}$  CHs are deployed in the first step, respectively. We have the following cases:

- (1) If both deployments satisfy the connectivity requirement, i.e.  $n_{RN}^{l1} > n_{RN}^{l2} \geq N_{RN}^{w\{\min\}}$ , no RNs are needed for the second step, and the argument holds.
- (2) If the 2<sup>nd</sup> deployment does not meet the connectivity requirement, while the 1<sup>st</sup> one does, then  $n_{RN}^{c2} \leq N_{RN}^{w\{\min\}} - n_{RN}^{l2} \leq n_{RN}^{l1} - n_{RN}^{l2}$  RNs are needed to be deployed to compensate for the density in the sparse areas, and the argument holds.
- (3) If both deployments do not satisfy the connectivity requirement, then deploy  $n_{RN}^{c1}$  and  $n_{RN}^{c2}$  RNs to satisfy the connectivity requirements according to formulas (36)-(41), respectively. As illustrated in Fig. 13, a sensing field is partitioned into  $A_1, A_2$  and  $A_3$  as in Section 4. The area  $A_2$  is further cut into three parts  $B_1, B_2$ , and  $B_3$ .  $B_1$  is the  $B$  area for the 1<sup>st</sup> deployment (formula (33)),  $B_1$  and  $B_2$  together are the  $B$  area for the 2<sup>nd</sup> deployment, and  $B_3$  is the rest of  $A_2$ . The expected number of RNs in  $B_1$  is the same for both deployments since both deployments just meet the connectivity requirement. The expected number of RNs in  $B_2$  of the 1<sup>st</sup> deployment is not less than that of the 2<sup>nd</sup> deployment because the 2<sup>nd</sup> deployment just meets the connectivity requirement while the 1<sup>st</sup> one provides better connectivity. The expected number of RNs in  $B_3$  of the 1<sup>st</sup> deployment is again not less than that of the 2<sup>nd</sup> deployment as both deployments provide good connectivity in the first place and  $n_{RN}^{l1} > n_{RN}^{l2}$ . For a similar reason, the expected number of RNs in the area  $A_1$  and  $A_3$  of the 1<sup>st</sup> deployment is not less than that of the 2<sup>nd</sup> deployment. Summing the numbers of RNs in the three parts, the argument holds.

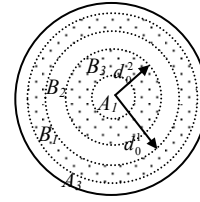


Fig. 13. Comparison of two hybrid deployments

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